# Proper Valuation of Perpetuities in an Inflationary Environment without Real Growth

Ignacio Vélez–Pareja
Universidad Tecnológica de Bolívar
Cartagena, Colombia
ivelez@unitecnologica.edu.co

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## Abstract

We examine the proper valuation of perpetuities without real growth. The case of a "pure" non growing perpetuity (zero real growth and zero inflation) is of academic interest but in practice it might be difficult to find. The findings contradict what is generally accepted in the literature. In particular we examine the textbook formula for calculating the value of a perpetuity.

When working with perpetuities we are in presence of a Chinese box: we have found that when working with perpetuities in a scenario of non zero inflation and zero real growth value increases with inflation. On the other hand, the textbook formula for calculating a non growing perpetuity in the same scenario under values the value of the perpetuity by relevant amounts.


**JEL codes**: D61, G31, H43

**Key words or phrases:** WACC, perpetuities, terminal value, tax savings

## Introduction

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When working with perpetuities we are in presence of a Chinese box: we have found that when working with perpetuities in a scenario of non zero inflation and zero real growth value increases with inflation. On the other hand, the textbook formula for calculating a non growing perpetuity in the same scenario under values the value of the perpetuity by relevant amounts.

The current literature lists the original Modigliani and Miller (M&M, 1958) formulation of the value of a non growing perpetuity that calculates the value of a non growing perpetuity as the Free Cash Flow FCF, divided by the nominal cost of capital. In the M&M world there is no inflation, hence in the case of a non growing perpetuity the discounting is done with a real interest rate. Copeland et al. (2000, p. 282) do not make explicit if the discount and growth rates are nominal or real. Brealey and Myers, (2003, p. 37) use the M&M formula without specifying that the discount rate should be in real terms. Copeland and Weston (1992, p. 440-441), Penman (2001, p. 108-109, 205-206) and Ross et al. (1999, p. 80-81) and Ogden, et al. (2003, p. 288) do not specify if the discount rate is real or nominal. Bradley and Jarrel (B&J, 2003) and Vélez-Pareja and Tham (VP-Th, 2005) in independent works point out that the M&M case applies only when nominal growth is zero, which means that inflation and real growth are zero. In their paper, B&J propose a Zero-Growth-Model model that complies with the condition of neutral inflation condition and zero Net Present Value, NPV. VP&Th propose a model that considers combinations of real growth and inflation at zero and non zero levels. This model is not inflation neutral.

In this work we conclude that models that deviate from the M&M conditions (zero real growth and zero inflation) show inconsistencies, including the one developed by B&J.

## The Case

In practice we should not expect a world with zero inflation, hence we restrict our case to non zero inflation and zero growth. When inflation is zero, we say we are in presence of a real Weighted Average Cost of Capital, WACC; when we have inflation, we say we have a deflated WACC. As will be shown, under inflation they are different: in the first case, real WACC is invariant with inflation; in the second case the size of the deflated WACC changes with inflation.

The FCF increases due to two drivers: one is the inflation present in the economy and another is the real growth due to increase in the volume of units (goods or services) sold. In the first case, the firm does not need to increase its level of assets. In the second case we assume that real growth is achieved when we increase the level of assets. This implies that we assume that the firm reached a point where the production capacity is fully attained.

We assume that the real growth rate is 0 and the FCFs are in perpetuity, and the leverage D% is constant. We can illustrate this situation with a simple example below.

## The Calculation of the Value of a Perpetuity

The first idea we should present is the Fisher relation that will help us in simplifying some expressions.

Fisher equation says:

$$1 + \text{nominal rate} = (1 + \text{real rate})\times(1 + i) \quad (1a)$$

Where i is the inflation rate.

From this expression we can derive the following:

$$\text{Nominal rate} - i = \text{Real rate} \times (1 + i) \quad (1b)$$

$$\text{Real rate} = (\text{Nominal rate} - i)/(1 + i) \quad (1c)$$

Real rate might be understood as the basic input data for a nominal rate or a deflated rate. Usually composed rates for instance, a rate that is composed of a rate of interest (say risk free rate) plus a risk component the deflated rate and the real rate might differ.

The mathematical calculation of the value of a perpetuity is as follows:

1. For a non growing perpetuity

$$TV^{L} = \frac{FCF_{N}}{DR} \quad (2)$$

Where $FCF_N$ is the free cash flow at the end of the forecasting period, DR is discount rate and $TV^L$ is the value of the perpetuity in period N. This expression implies zero inflation and zero real growth. It is known as M&M model for solving the value of a perpetuity.

2. For a growing perpetuity value is

$$TV^{L} = \frac{FCF_{N} \times (1+g)}{DR - g} \quad (3)$$

When we have perpetuities with non zero inflation and zero real growth the FCF grows only due to inflation. There is no need of extra investment[1]. From (1c) we have

$$TV^{L} = \frac{FCF_{N} \times (1+i)}{WACC - i} = \frac{FCF_{N}}{WACC_{deflated}} \quad (4)$$

Where WACC is the nominal WACC, i is the inflation rate, $WACC_{deflated}$ is the deflated WACC with the current inflation rate and other variables have been defined above. If we wish to use the format of the M&M model, we should use the right side hand of (4).

This is the "correct" model (Bradley and Jarrel (2003) call this model the Zero-Real-Growth model. However, in our formulation (see (3) and (4) above) the value calculated with this formulation is not inflation neutral. The value increases with inflation. This can be understood with the APV method. The unlevered value is constant, but the value of the TS is not constant. This last part of the APV is not inflation neutral and that is the reason why the value of the perpetuity, against common knowledge, increases with inflation. In this case, inflation creates value! Perpetuities, we have learned to say, are a Chinese Box.

We calculate the value of perpetuity using the Adjusted Present Value, APV and the Capital Cash Flow (see Appendix B). In order to calculate the APV we have to define the value of the TS. The TS grows at the same growth rate of the FCF, starting in period 2 of the perpetuity. Hence, the value of the perpetuity of TS is

$$\frac{T \times Kd \times D}{Ku - i} = \frac{T \times Kd \times D}{Ku_{deflated}(1+i)} \quad (5a)$$

The unlevered value of a perpetuity is

[1] We are using a general expression for WACC for perpetuity as a function of Ku, the unlevered cost of equity, the cost of debt, the corporate tax rate and the perpetual leverage. In this case, WACC = Ku – Kd×T×D%.

$$\frac{FCF_N \times (1+i)}{Ku - i} = \frac{FCF_N}{Ku_{deflated}} \quad (5b)$$

In Vélez-Pareja and Tham (2005) they derive the formulation for deflated and nominal WACC for perpetuity when there is no real growth but nonzero inflation and according to the discount rate for the tax savings TS. We use the formulation when the discount rate for the tax savings is Ku, as follows:

Table 1. Formulas to be used in valuing a perpetuity assuming the discount rate of TS as Ku

| | |
|---|---|
| $WACC_{deflated}$ | $Ku_{deflated} - \frac{T \times Kd \times D\%}{1+i}$ (5) |
| WACC | $Ku - T \times Kd \times D\%$ |
| Unlevered value | $\frac{FCF_N \times (1+i)}{Ku - i}$ |
| $V^{TS}$ | $\frac{T \times Kd \times D}{Ku - i} = \frac{T \times Kd \times D}{Ku_{deflated}(1+i)}$ |
| APV | $\frac{FCF_N \times (1+i) + T \times Kd \times D}{Ku - i}$ |

For illustration the calculation of value for the two methods is done assuming that the discount rate for the tax savings, TS is Ku. We compare these results with the typical approach of calculating a non growing perpetuity (Bradley and Jarrel (2003) call this the Zero-Nominal-Growth model) as

$$TV^L = \frac{FCF_N}{WACC} \quad (6)$$

Where WACC is nominal WACC.

Now we construct a simple example with the following input data:

Table 5a. Input data for example

| | |
|---|---|
| Real interest rate, $i_{real}$ | 3.0% |
| Inflation rate, i | 4.0% |
| Real cost of unlevered equity, Kur | 12.0% |
| Tax rate, T | 35.0% |
| Leverage in perpetuity, D% | 30.0% |
| Free cash flow, FCF | 100 |

From these input data we calculate the following values:

Table 5b. Values calculated from input data

| | |
|---|---|
| $R_f = (1+i_{real})\times(1+i)-1$ | 7.1% |
| Risk premium in Kd | 5.0% |
| Kd = Rf + Risk premium | 12.1% |
| $Kd_{deflated} = (1+Kd)/(1+i)-1$ | 7.8% |
| $Ku=(1+Ku_{real})\times(1+i)-1$ | 16.5% |
| Nominal WACC = Ku – Kd×T×D% | 15.21% |

In the next table we show the values calculated using the FCF, the WACC at perpetuity and the APV and compare them with the traditional textbook formula. These calculations are made under the assumption that the discount rate for the TS ψ, is Ku, the cost of unlevered equity.

Table 6. Value of the perpetuity using the FCF and the APV methods compared with the textbook method

| Inflation | $TV^L = \frac{FCF_N \times (1+i)}{WACC - i}$ | Unlevered value $=FCF/Ku_{deflated}$ | Value of TS $\frac{T \times Kd \times D}{Ku - i}$ | Total Value | $TV^L = \frac{FCF_N}{WACC}$ |
|---|---|---|---|---|---|
| 0% | 896.1 | 833.3 | 62.7 | 896.1 | 896.1 |
| 1% | 904.1 | 833.3 | 70.7 | 904.1 | 821.6 |
| 2% | 912.0 | 833.3 | 78.7 | 912.0 | 758.5 |
| 3% | 920.0 | 833.3 | 86.7 | 920.0 | 704.4 |
| 4% | 928.0 | 833.3 | 94.6 | 928.0 | 657.6 |
| 5% | 935.9 | 833.3 | 102.6 | 935.9 | 616.6 |
| 6% | 943.8 | 833.3 | 110.5 | 943.8 | 580.3 |
| 7% | 951.7 | 833.3 | 118.4 | 951.7 | 548.2 |
| 8% | 959.6 | 833.3 | 126.3 | 959.6 | 519.4 |
| 9% | 967.5 | 833.3 | 134.1 | 967.5 | 493.4 |
| 10% | 975.3 | 833.3 | 142.0 | 975.3 | 470.0 |

Firstly we have to call the attention is that the value of a non real growing perpetuity on an inflationary world is not inflation neutral. This is clear when we observe the value of the TS. As inflation increases the cost of debt increases and that creates value. The other issue that has to be observed is that when we use the nominal WACC value decreases.

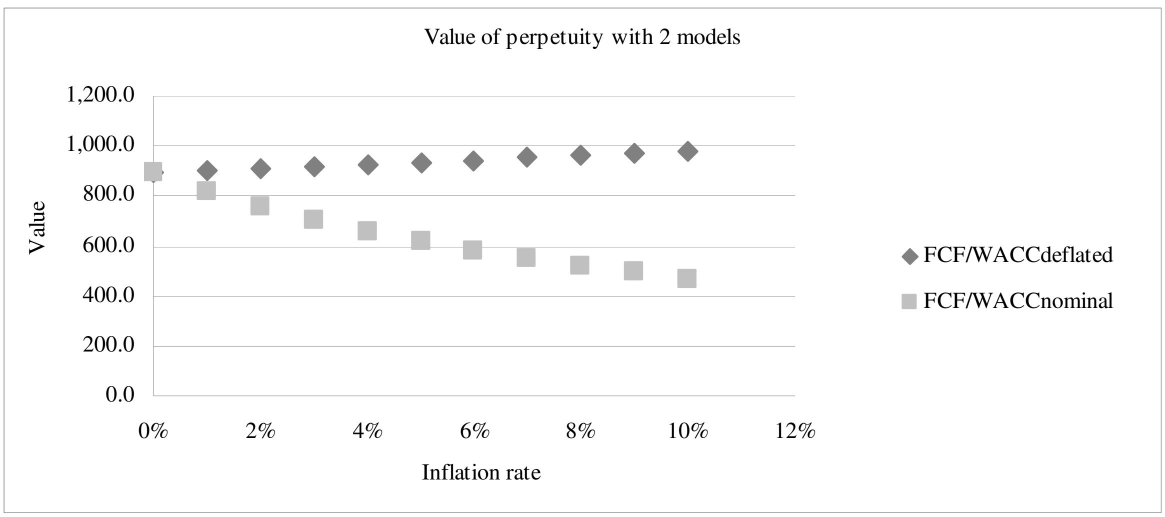


Exhibit 1. Value of perpetuities with two methods

Another way to see this is to test the undervaluation of the perpetuity when nominal WACC is used. This can be seen in the next table where we present each model.

Table 7. Measure of undervaluation when using nominal WACC to value a perpetuity with inflation

Inflation rate

| | -29.14% | 0% | 2% | 4% | 6% | 8% | 10% |
|---|---|---|---|---|---|---|---|
| | 10% | 0.0% | -16.1% | -27.8% | -36.7% | -43.7% | -49.4% |
| | 20% | 0.0% | -16.4% | -28.5% | -37.6% | -44.8% | -50.6% |
| | 30% | 0.0% | -16.8% | -29.1% | -38.5% | -45.9% | -51.8% |
| D% | 40% | 0.0% | -17.2% | -29.9% | -39.5% | -47.0% | -53.1% |
| | 50% | 0.0% | -17.7% | -30.6% | -40.5% | -48.3% | -54.6% |
| | 60% | 0.0% | -18.1% | -31.4% | -41.6% | -49.6% | -56.1% |
| | 70% | 0.0% | -18.6% | -32.3% | -42.8% | -51.0% | -57.7% |

Observe that even for low inflation rates the undervaluation is significant. For instance, for a yearly inflation of 2% and a D% of 40% the undervaluation is as high as 30%.

In Exhibit 2 we show this undervaluation for our example.

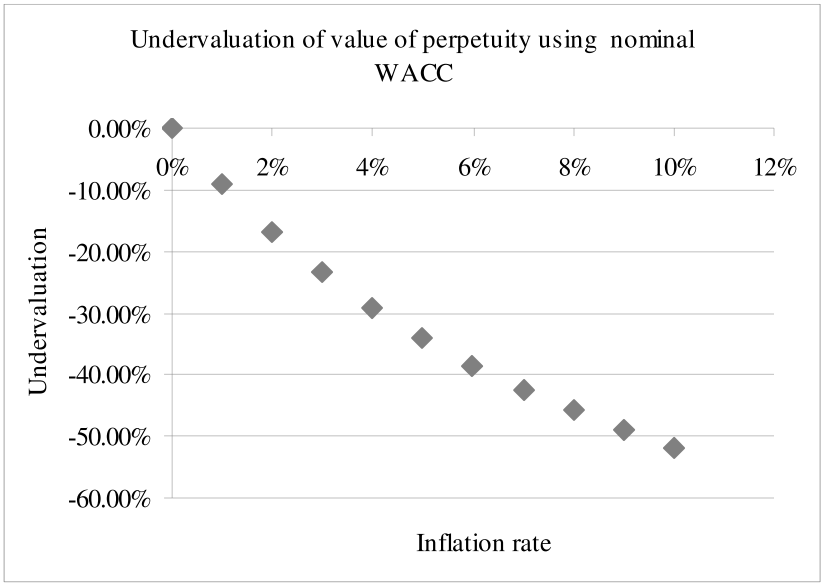


Exhibit 2. Undervaluation when using the textbook formula for non growing perpetuities.

In the previous table and exhibit we show a situation similar to the one presented by Bradley et al (2003), but worse. B&J propose a model based on a real return equal to real WACC. This model gives zero, a constant value, and is neutral inflation. However, when tested for consistency (for instance, when calculating the value of the perpetuity with a model like Adjusted Present Value (APV), it is not constant, is not inflation neutral and is different to the value calculated by them.

B&J model says

TV=Ko×r×((1+i))/((W-i) )=Ko (7)

Where TV is the value of the perpetuity, Ko is initial investment, r is the real return of capital and equal to w, the real WACC, W is the nominal WACC and i is inflation.

The value of the perpetuity could be calculated using the APV we have:

TV=(Ko×r)/(Ku-i) + (Kd×T×D%×TV)/(Kd-i) (8)

Where Ku is the unlevered cost of equity, Kd the cost of debt, D% the leverage in perpetuity and other variables have been defined above.

Using the same data they have in the paper, plus additional data for Kd, D% and Ku, we have the following:

Table 8. Input data

| | |
|---|---|
| Ko initial invested capital | 100 |
| Real WACC w = r return on invested capital | 7.0% |
| FCF | 7.0 |
| Inflation rate | 4.0% |
| Nominal WACC | 11.3% |
| Value | 100 |
| Kdr | 6.0% |
| Kd | 10.2% |
| T | 35.0% |
| D% | 30.0% |
| Ku real | 7.821% |
| Ku | 12.13% |

On the other hand, the values calculated using the two methods are:

| Inflation | Bradley Approach | Unlevered Value | $PV^{TS}$ | V= APV |
|---|---|---|---|---|
| 0% | 100.0 | 89.50 | 10.50 | 100.00 |
| 2% | 100.0 | 89.50 | 14.49 | 103.99 |
| 4% | 100.0 | 89.50 | 18.63 | 108.13 |
| 6% | 100.0 | 89.50 | 22.95 | 112.45 |
| 8% | 100.0 | 89.50 | 27.44 | 116.94 |

As can be seen, the only inflation rate for which the two methods coincide is when inflation is zero.

In (4) we have the value of a perpetuity with non zero inflation and zero real growth. This is the correct model in that scenario. On the other hand, FCF/WACC is the textbook formula for non growing perpetuities.

In summary, the traditional approach (Modigliani and Miller, 1958) is insensitive to inflation only when G, the nominal growth rate is zero and this means zero inflation and real growth. When there is inflation the value of a non growing perpetuity is not inflation neutral. This finding is similar to the one found by Bradley and Jarrel (2003). The main difference is that they consider that the value of perpetuity with the proper formulation is inflation neutral and we have shown it is not. This is against current and common knowledge, but as we have explained above, the value of the TS is not inflation neutral and the value of the perpetuity is not inflation neutral, as shown with the APV, above.

## Concluding Remarks

We have shown that the value of a non growing perpetuity in an inflationary environment depends on inflation and the formulation is different from what is consecrated in the financial literature is inappropriate. The "pure" non growing perpetuity is a particular case where the deflated WACC is the real WACC.

When we have inflation the FCF of a "non growing" perpetuity grows at the inflation rate and the WACC to be used to calculate the value of this perpetuity is the deflated WACC if we wish to use the format of the M&M model.

The proper formulation under inflation is

- $TV^L = \frac{FCF_N \times (1+i)}{WACC - i} = \frac{FCF_N}{WACC_{deflated}}$ for non real growing perpetuity, this is g = 0 and non zero inflation.

The typical textbook formulation that uses the M&M model with the nominal WACC as the discount rate without taking into account that the M&M model is correct only when inflation and real growth are zero, undervalues the perpetuity. The level of undervaluation is high even in economies with low level of inflation.

The main findings of this work are:

1. The textbook formula for a non growing perpetuity is misleading and is based on the M&M model that is correct for a scenario of no inflation and no real growth.
2. The correct model (with the scenario of non zero inflation and zero real growth) is not inflation neutral and the value of the tax savings (that is part of the perpetuity value, increases with inflation.
3. If we use correct methods to value perpetuities, we obtain results that are counter evident such as that inflation creates value, which makes no sense in all cases. One thing is that we try to understand why occurs, and another is to endorse the models.
4. We have shown that there are models that try to replicate the M&M solution to perpetuities with zero growth and zero inflation. However, once we introduce inflation in the model, it collapses.

What to do in real life? Perhaps the best solution might be to use the M&M model with zero growth and zero inflation rate. This is conservative, but consistent.

**Bibliographic references**

Bradley, Michael H. and Jarrell, Gregg A., 2003, "Inflation and the Constant-Growth Valuation Model: A Clarification" (February). Simon School of Business Working Paper No. FR 03-04. http://ssrn.com/abstract=356540

Brealey, Richard and Stewart C. Myers, 2003, Principles of Corporate Finance, 7th edition, McGraw Hill-Irwin, New York.

Copeland, Thomas E., Koller, T. y Murrin, J., 2000, *Valuation: Measuring and Managing the Value of Companies*, 3rd Edition, John Wiley & Sons, (July 28).

Copeland, Thomas and Fred Weston, 1988, "Financial Theory and Corporate Policy". Addison-Wesley Publishing Company, tercera edición 1988

Miller, M. H. y F. Modigliani, 1961, Dividend Policy, Growth and the Valuation of Shares, *The Journal of Business*, V. 34, No. 4, P. 411-433 (Oct).

Ogden, J.P., F.C. Jen and P.F. O'Connor, 2003, *Advanced Corporate Finance. Policies and Strategies*, Prentice Hall, New Jersey.

Taggart, R., 1991, Consistent Valuation and Cost of Capital Expressions with Corporate and. Personal Taxes. *Financial Management*, Autumn, pp. 8-20

Tham, Joseph and Ignacio Velez-Pareja, 2002, "An Embarrassment of Riches: Winning Ways to Value with the WACC" (November). Available at SSRN: http://ssrn.com/abstract=352180 or DOI: 10.2139/ssrn.352180.

Tham, Joseph and Vélez-Pareja, Ignacio, *Principles of Cash Flow Valuation. An Integrated Market Approach*, Academic Press, 2004.

Velez-Pareja, Ignacio and Joseph Tham, 2005, "*WACC, Value of Tax Savings and Terminal Value for Growing and Non Growing Perpetuities*" (August 30,). http://ssrn.com/abstract=789025

**Appendix A**

Why deflated WACC is not identical to real WACC

$$WACC = Ku - Kd \times T \times D\% \quad (1)$$

$$WACCr = Kur - Kdr \times T \times D\% \quad (2)$$

Ku is calculated from real Ku as follows

$$Ku = (1+Kur) \times (1+i)-1 \quad (3)$$

Where Ku is the unlevered cost of equity, Kur is the real cost of unlevered equity and i is inflation rate.

$$Kd = (1+Kdr) \times (1+i)-1 \quad (4)$$

Where Kd is nominal cost of debt and Kdr is the deflated cost of debt (in theory, Kd is the risk free rate and hence Kdr is the real rate of interest).

If we replace (3) and (4) in (1) and deflate the result using the inflation rate, we do not obtain (2). Let us see it.

$$Ku - Kd \times T \times D\% = (1+Kur) \times (1+i)-1 - ((1+Kdr) \times (1+i)-1) \times T \times D\% \quad (5a)$$

$$Ku - Kd \times T \times D\% = Kur + i + Kur \times i - (Kdr + i + Kdr \times i) \times T \times D\% \quad (5b)$$

Deflated WACC is

$$(1+Kur + i + Kur \times i)/(1+i) - (Kdr + i + Kdr \times i) \times T \times D\%/(1+i) - 1 \quad (6a)$$

$$1 + Kur - (Kdr + i + Kdr \times i) \times T \times D\%/(1+i) - 1 \quad (6b)$$

$$Kur - Kdr \times T \times D\% + i \times T \times D\%/(1+i) \quad (6c)$$

As can be seen this formula is quite different from (B2). The difference is $i \times T \times D\%/(1+i)$ and this, in general is not zero. It is zero only when inflation rate is zero and in that case we say that deflated WACC is real WACC.

**Appendix B**

Complete set of formulas for valuing perpetuities

For calculating value using the FCF we have the deflated and nominal WACC.

Table B1a $WACC_{deflated}$ depending on the value of ψ when $g_r = 0$ i > 0

| | |
|---|---|
| ψ = Ku | $Ku_{deflated} - \frac{T \times Kd \times D\%}{1+i}$ (5) |
| ψ = Kd | $Ku_{deflated} - \frac{Ku_{deflated} \times Kd \times T \times D\%}{Kd_{deflated} \times (1+i)} = Ku_{deflated}\left(1 - \frac{Kd \times T \times D\%}{Kd - i}\right)$ (6) |

The nominal WACC is

Table B1b $WACC_{nominal}$ depending on the value of ψ $g_r = 0$ i > 0

| | |
|---|---|
| ψ = Ku | $Ku - T \times Kd \times D\%$ (7) |
| ψ = Kd | $Ku - \frac{Ku_{deflated} \times Kd \times T \times D\%}{Kd_{deflated}}$ (8) |

We also can calculate the value of perpetuity using the Adjusted Present Value, APV and the Capital Cash Flow. In order to calculate the APV we have to define the value of the TS. The TS grows at the same growth rate of the FCF, starting in period 2 of the perpetuity. Hence, the value of the perpetuity of TS is

Table B2. $V^{TS}$ formulation for two values of ψ, the discount rate of TS when $g_r = 0$ i > 0

| | |
|---|---|
| ψ=Ku | $\frac{T \times Kd \times D}{Ku - i} = \frac{T \times Kd \times D}{Ku_{deflated}(1+i)}$ (9) |
| ψ=Kd | $\frac{Kd \times T \times D}{Kd - i} = \frac{Kd \times T \times D}{Kd_r (1+i)}$ (10) |

The unlevered value of a perpetuity is

$$\frac{FCF_N \times (1+i)}{Ku - i} = \frac{FCF_N}{Ku_{deflated}}$$

Then the APV is shown in the next table.

Table 3. APV calculation depending on the discount rate of the TS when $g_r = 0$ i > 0

| | |
|---|---|
| ψ=Ku | $\frac{FCF_N \times (1+i) + T \times Kd \times D}{Ku - i}$ (11) |
| ψ=Kd | $\frac{FCF_N \times (1+i)}{Ku - i} + \frac{Kd \times T \times D}{Kd - i}$ (12) |

The Capital Cash Flow for period N+1 is

$$FCF_N \times (1+i) + TS = FCF_N \times (1+i) + T \times Kd \times D \qquad (13)$$

The value of the perpetuity for CCF is the present value of the CCF at the WACC for the CCF.

Table 4. Value of the perpetuity using the Capital Cash Flow when $g_r = 0$ $i > 0$

| | |
|---|---|
| $\psi$ = Ku | $\frac{FCF_N \times (1+i) + T \times Kd \times D}{Ku - i}$ (14) |
| $\psi$ = Kd | $\frac{FCF_N \times (1+i) + Kd \times T \times D}{Ku - (Ku - Kd) \times T \times D\% - i}$ (15) |