The dependence of prices on labour-values

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Abstract

It is frequently believed, in a quite schizophrenic fashion, that a theory of value must just solve the question of “relative prices” (a microeconomic problem), being mainly the theory of money the piece needed for determining the absolute or monetary level of prices (a macroeconomic problem). But on the one hand, the determination of the level of prices is theoretically prior to any consideration of the money market, whereas on the other hand no theory of value can aspire to be complete without the determination of the absolute level of values. It will be shown in this paper that only the Labour theory of value (LTV) can perform both tasks, thus giving *completeness* and *unity* to economic theory. It is frequently acknowledged that, as labour is—or “is treated as”, as the critics of the LTV say—the only factor of production of value (even if it is just one of the several factors producing wealth), the determination of prices is independent of demand in the long run. However, prices are *not* determined by technical or physical data plus wages, contrarily to what is commonly thought. It is only the couple formed by “relative prices and the rate of profit” that is determined by them, as well as the couple “relative values and the rate of surplus value”. By contrast, it can be shown that absolute prices crucially depend on, and in fact are determined by, absolute values, what will be illustrated in this paper by means of a numerical example of an economy with only two industries, where for example halving the quantity of labour or value reduces the level of prices by a 50%. The path of thought that will lead us to these conclusions requires previous clarifications of the several and frequently poorly understood Marxian concepts of value (and price), and a new view on the question of the transformation of “value prices” (Marx’s term) into “production prices”, both of which will be developed simultaneously with the main line of argument.

Resumen

Frecuentemente se cree, de un modo bastante esquizofrénico, que una teoría del valor resuelve la cuestión de los "precios relativos" (un problema microeconómico), siendo principalmente la teoría del dinero la pieza necesaria para la determinación del nivel absoluto o monetario de los precios (un problema macroeconómico). Pero, por una parte, la determinación del nivel de precios va teóricamente antes que cualquier consideración del mercado monetario, mientras que por otro lado, ninguna teoría del valor puede estar completa sin la determinación del nivel absoluto de los valores. Se

mostrará en este trabajo que solo la teoría del valor-trabajo (LTV en inglés) puede resolver ambas tareas, dando así integridad y unidad a la teoría económica. Con frecuencia se reconoce que, como el trabajo es o "representa", como dicen los críticos de la LTV, el único factor de producción de valor (incluso si es solo uno de los varios factores de producción de riqueza), la determinación de los precios es independiente de la demanda a largo plazo. Sin embargo, los precios no vienen determinados por los datos técnicos o físicos, más los salarios, contrariamente a lo que comúnmente se piensa. Es solo la combinación formada por "los precios relativos y la tasa de ganancia" la que está determinada por los precios, al igual que la combinación "valores relativos y tasa de plusvalía". Por el contrario, se puede demostrar que los precios absolutos dependen de manera crucial, y de hecho están determinados por los valores absolutos, tal y como se muestra en este trabajo por medio de un ejemplo numérico de una economía con solamente dos sectores, donde, por ejemplo, reduciendo a la mitad el trabajo o el valor se reduce el nivel de precios en un 50%. La línea de pensamiento que nos lleva a estas conclusiones requiere algunas aclaraciones previas de varios conceptos marxistas con frecuencia mal entendidos como el valor (y el precio), y una nueva visión de la transformación de "precios de valor" (expresión de Marx) en "precios de producción", conceptos que se desarrollarán simultáneamente junto con la línea de razonamiento principal de este trabajo.

Introduction: the crucial role of absolute values

Almost all critics of the labour theory of value (LTV), particularly Neoclassicists and Sraffians, share a rejection of the concept of "absolute value" (that at least comes back to Bailey, 1825)[1] and probably a misunderstanding of its role in the Marxian theory of value. Historically, the rejection of the concept of absolute value has not always been a rejection *per se*. For instance, the founder of the theory of General equilibrium, Léon Walras, although thinking that "value is essentially relative", was convinced that "to be sure, behind relative value, there is something absolute" (Walras, 1926, p. 188). Indeed, Walras was opposed to the idea that labour is the foundation or cause of value, proposing instead the *rareté* as an alternative, subjective principle, an "absolute and subjective" phenomenon (ibid., p. 178). However, his rejection manifests itself in that he declares to prefer to avoid using what we may call the "absolute" point of view; this is why, after having written that "in a state of general equilibrium *each commodity has only one value in exchange in relation to all other commodities on the market*", he adds that "this way of putting" is "perhaps too likely to be constructed as if absolute value were meant, and, therefore, it is preferable to describe the phenomenon in question in terms of the theorem of general equilibrium (§ 111) or in terms of the analytical definition of exchange (§ 131)." (p. 178).

Joseph A. Schumpeter clearly realized that the concept of absolute value was the "central concept" in Marx's theory (1954, p. 598). Although, according to him, Marx's absolute value was "but Ricardo's real value, fully worked out and fully made use of", he added that Marx not only "actually went through with the idea of an absolute value of things", but he was in fact "the only author who ever did" (ibid., pp. 597-8). Therefore, the features observed by Schumpeter in Ricardo's absolute values must *a fortiori* be predicated of Marx's, being also the latter "capable of being compared, added up, and of increasing and decreasing simultaneously", whereas all of this would be "impossible so long as exchange *value* was defined simply as exchange *rate*" (ibid., p. 591).

Of course, Karl Marx was well aware that "an *intrinsic* value, i.e. an exchange-value that is inseparably connected with the commodity, *inherent* in it, seems a contradiction in terms"; this is so because it "appears first of all" as a "quantitative relation" or proportion "in which use-values of one kind exchange for use-values of another kind", and also because this relation "changes constantly with time and place", and "hence exchange-value appears to be something accidental and purely relative" (Marx, 1867, p. 126; our emphasis). However, in coherence with

[1] Bailey was a precedent. He said that "value denotes (…) nothing positive or intrinsic, but merely the relation in which two objects stand to each other as exchangeable commodities" (quoted in Dobb, 1973, p. 99). He attacked "even Malthus for sponsoring the notion of 'invariable, absolute, natural' value (in his *Measure of Value*) by contrast with 'nominal or relative value'" (in Dobb, ibid., p. 100).

Marx's ideas about the relation between essence and appearance, it is no surprise that he thought that relative values were just an appearance, a "semblance", whereas "the determination of the magnitude of value by labour-time is therefore a secret hidden under the apparent movements in the relative values of the commodities" (ibid., p. 168).

In contrast with these ideas, it is well known that in neoclassical theory "no conception of 'absolute' value (…) is either relevant or necessary"; on the contrary, authors belonging to this tradition in economics are "accustomed to thinking of the basic problem of price theory as being the determination of sets of relative prices, with any consideration of 'absolute' value being confined to problems in monetary theory and the determination of the overall price level" (Eatwell, 1987, p. 3). Certainly, this approach is not without problems, and its supporters have to acknowledge, beginning with Walras's troubles about counting equations, what Arrow and Hahn call "offsetting complications", for "the system of equations has only $n-1$ variables, a point that Walras expressed by selecting one commodity to serve as *numeraire*, with the prices of all commodities being measured relative to its price". (Arrow and Hahn, 1971, p. 4)[2].

But it is curious that even most critics of neoclassical economics arrive at results that contradict the position they seem to be defending. For example, after recalling us that, in his opinion, both Ricardo and Marx were unsuccessful in their effort to establish the foundations of absolute value, Eatwell shows his agreement with the relative prices perspective. This is why he writes that "the data of classical theory can be used to determine the rate of profit, as Sraffa (1960) has shown", or that "the rate of profit and the rates at which commodities exchange must be determined *simultaneously*"; from which he concludes that the determination of he rate of profit "cannot be *sequential*—first specifying a theory of value and then evaluating the ratio of surplus to capital advanced by means of that *predetermined* theory of value". (Eatwell, 1987, p. 4; our emphasis).

However, what most authors have not realized, as Eatwell either, is that the determination of the "rates at which commodities exchange" is *not* yet the *determination* of prices, since the solution of the *relative* prices side of the problem is not the solution of its *absolute* prices side, so that the theory of prices remains incomplete until the latter is solved. In distinguishing carefully between these *two* different aspects of the problem, we understand better why the "transformation problem" is not an "intrinsically unimportant problem", as Steedman seems to believe (1977, p. 29); in fact, the relationship between absolute and relative values is the core of the problem of the "overall level of prices" (see section 9) and is, therefore, the crucial link between labour-values and money[3], and between the theory of value and the

[2] Other, more sophisticated complications appear in other areas of study. See several examples in the review made by Willenbockel (2005) in the field of the normalization of prices in oligopoly GE models.
[3] This link is usually interpreted in an artificial way: "In this model [Walrasian General Equilibrium] the absolute level of prices cannot be determined. General Equilibrium theorists have adopted the device of choosing arbitrarily the price of one commodity as *numéraire* (or unit of account) and express all other

theory of money, i.e. between microeconomics and macroeconomics. However, before going deeper into the transformation problem, we have to deal with some preliminary clarifications.

1. What values are we really speaking of

Let us begin this section by having a look at Tables 1 and 2, that help us to explain why raising the question of what values are we really speaking of, when one places oneself in the LTV, is not a joke.

Value = Labour (Classicists)	"Natural" prices
	(Market prices)
Value = Utility (Neoclassicists)	"Equilibrium" prices
	(Disequilibrium prices)

Table 1: Values and Prices in Classical and Neoclassical economists

For both Classical and the first Neoclassical economists, the relationship between values and prices seemed to be of not so different nature, at least in as much as both in principle admitted the existence of "absolute values" (even if modern Neoclassicists do increasingly tend to avoid even the mere mention of this concept). Either consisting of quantities of labour or of utility, absolute values were acknowledged but, as said, these authors preferred to focus on relative prices, especially those that are theoretically most relevant: "natural" or "equilibrium" prices. In fact, an increasing adoption of the equilibrium perspective only have lead Neoclassical authors to make actual market prices—which are always disequilibrium prices—practically disappear; as they tend to confine themselves to equilibrium prices (as if they were the sole and ultimate target of analysis).

By contrast, Marx's treatment of prices was much more developed and complete, as can be seen in Table 2. In the first step, we can observe that his "intrinsic", "inherent" or "absolute" values are *labour*-values indeed, as corresponds to a *labour* theory of value; whereas "relative values", or "prices", are exchange-values or *money*-values: values that have a different form ("form of value"). Marx thinks that both the substance and the *intrinsic measure*[4] of values

prices in terms of the price of the *numéraire*. With this device, prices are determined only as ratios; each price is given relative to the price of the *numéraire* (…). If we assign unity to the price of the *numéraire*, we attain equality of the number of simultaneous equations and unknown variables (…). However, the absolute prices are still not determined: they are simply expressed in terms of the *numéraire*. This indeterminacy can be eliminated by the introduction explicitly in the model of a money market, in which money is not only the *numéraire* but also the medium of exchange and store of wealth." (Koutsoyiannis, 1979, p. 488).

[4] "Money as a measure of value, is the phenomenal form that must of necessity be assumed by that measure of value which is immanent in commodities, labour-time" (Marx, 1867, p. 51).

are labour but their *necessary form of expression* is money, so that values have to be expressed as prices[5]. When dealing with this first aspect of the question, one might want to speak, for the sake of simplicity, of "a-values" and "b-values", being the former expressed in hours, minutes, etc., and the latter in euros, dollars, etc.

"a-values" **(Intrinsic, Absolute, or Labour values)**	**"b-values" (or prices)** **(Exchange, Relative, or Money values)**
a.1. "labour value": v	b.1. "value price", or "market value": v_p
(a.2. "production price-value", in hours of labour): p_v	b.2. "production price": p
(a.3. "market price-value", in hours of labour): m_v	b.3. "market price": m

Table 2: Different meanings of Value and Price in a Marxian framework

The rationale for this is simple: it is true that Marx's theory of value distinguishes between a "substance" of value and a "form" of value, but these terms should not suggest that the former is *more important* than the latter; in fact, there is a perfect correspondence between the two, up to the point that "it was solely the analysis of the prices of commodities which led to the determination of the magnitude of value, and solely the common expression of all commodities in money which led to the establishment of their character as values" (Marx, 1867, p. 168). Thus, the *transition* from one "kind" of values into the other, from a-values to b-values or vice versa, is just a question of "translation"[6] that can be practically performed by using what is commonly called the "monetary expression of labour time"[7], e (i.e. using it for multiplying or dividing, as we will see later).

But this first, primary and visually "horizontal" distinction between a-values and b-values is not enough: it has to be supplemented by a "vertical" and more sophisticated distinction. The money expression of the unit value of any sort of commodity can and must be dissected into *three different magnitudes* of its "price" (and the same must be said of their labour content), according to the special meaning we are giving to the word *price* in every case. First of all, what

[5] A suggestive new interpretation of Marx's LTV is offered by Heinrich (2004), who prefers to call it a "monetary" theory of value instead of a "labour" theory. However, although Heinrich's ideas go in some aspects clearly beyond the traditional Marxian views, we believe more adjusted to Marx's thought to speak of his theory as a "labour-and-monetary" theory of value, which fits well by the way with the arguments and conclusions of this paper.

[6] For Marx, the price is just "the money-name of the labour realised in a commodity", even if "the name of a thing is something distinct from the qualities of that thing" (Marx, 1867, pp. 53-4).

[7] We think that a better name for it would be "monetary expression of *overall* labour time", and a suited name for its reciprocal, e^{-1}, might be the "*labour substance of the money price of social production*".

Marx aims to understand is ultimately the behaviour of *actual* or *market prices*: the vector m. Of course, no author ignores the existence of those prices, but only Marx has always market (disequilibrium) prices present, alongside his version of "equilibrium" prices, which he calls production prices, vector p. Secondly, for the first time in the history of economic thought, Marx highlighted the need to make *two successive steps* in studying which prices are the *regulators* of actual prices. He believed indeed that the *immediate* regulators of actual prices are p, the prices of production; but he also thought that the latter can *only* be fully understood when starting from their own regulators: "*value prices*"[8], or vector v_p. These value-prices are "proportional" (by factor e) to the labour quantities involved in the production of commodities.

With p Marx was of course accepting Smith's idea of the "invisible hand", in the sense that the search for maximum profit on the part of each capitalist puts in motion a general tendency towards the formation of prices that are the sum of the cost price plus the volume of profit that yields the same rate of profit for all industries. But the existence of prices of production does not cancel the existence of value prices; there is *no contradiction* between them as if their coexistence in the same world (both theoretical and practical worlds) were not possible. The pervasive idea from Böhm-Bawerk to Samuelson that such a contradiction exists is accepted even by some critics of the LTV that acknowledge the importance of Marx's contribution to economics and in particular to the theory of value. This is the case of Arrow and Hahn, who after pointing that Smith was in a sense "the creator of general equilibrium theory" conclude that "in some ways Marx came closer in form to modern theory in his schema of simple reproduction (*Capital*, Vol. II), studied in combination with his development of relative prices theory (Vols. I and III), than any other classical economist, though he confuses everything by his attempt to maintain simultaneously a pure labour theory of value and an equation of rates of return on capital" (Arrow and Hahn, 1971, p. 2). It is quite probable that these authors know that in "transformation (…) commodities, while still retaining their values, were *not* sold at relative prices proportional to these values"; but they seem to overlook the second part of Schumpeter's statement: that for Ricardo the latter amounted to "alterations of values", whereas for Marx "such deviations did not alter values but only redistributed them as between the commodities" (p. 597).

We can improve the presentation of the above ideas by means of a bit of matrix algebra[9]. Calling x the vector of unit outputs, v_p the vector of unit value-prices, and p the vector

[8] This term is Marx's, and its meaning is clear in the following paragraph: "Let us assume to start with that all commodities in the various spheres of production were sold at their actual values. What would happen then? According to our above arguments, very different rates of profit would prevail in the various spheres of production. It is, *prima facie*, a very different matter whether commodities are sold at their values (i.e. whether they are exchanged with one another in proportion to the value contained in them, at their value prices) or whether they are sold at prices which make their sale yield equal profits on equal amounts of the capitals advanced for their respective production." (Marx, 1867, p. 275)

[9] We will denote row vectors by adding an apostrophe to the symbol of the corresponding column vectors.

of unit production-prices, the meaning of such a "redistribution" is that even if at the commodity level $p \neq v_p$ in the general case, we would at the aggregate level always have:

$$p'x = v_p'x. \qquad (1).$$

Value prices are, as said, proportional to labour values. Since the *first* contention of the LTV is that new labour is the *single* factor of production of new *value*, or "value added" (even if the factors of production creating *wealth* are *many*), the most logical would be to begin with a *first* "quantitative definition" of values as those that contain a value added *proportional to the wages* paid for the purchase of this factor of production: the labour force which performs new labour. If competition did not force every firm to behave in a way than contributes to collect in their industry a mass of profit *tendencially* proportional to the entire capital *advanced* in that industry (i.e. the sum of constant and variable capital: $K_i = C_i + V_i$), at the general rate r, the inherent value under the value price would form prices containing the sum of both constant and variable capital *expended* in the period plus a profit *tendencially* proportional to variable capital only, as the competition among workers would force them to eventually accept, as their payment, the equivalent of identical fraction of the average working day in every industry, which would yield the same (general) rate of surplus value, s, for all industries.

In this way, value-prices reproduce, at the microeconomic level of analysis, what is the most basic idea at the macro level of analysis of the LTV: social labour, L, creates all new value; but the reproduction of the subjects who perform L only costs to capital the amount of the mass of wages it pays, V, which is just a fraction of the value added by workers—being the surplus value, S, equal to the difference $L - V$. Marx is not a methodological individualist. On the contrary, he believes necessary—before coming closer to how each capital fights against each other, at the *microeconomic* level, to get as big a part as possible from the common loot provided by the joint exploitation of the overall labour force—to start with value prices, v_p, prices that are required by the will to provide the analysis with a (methodologically prior) macro-social perspective.

2. The quantification of values and the rate of surplus value

A crucial aspect of what we are discussing in this paper is the quantitative definitions of the entire set of values and prices of Table 2 (we can group them in three couples: v-v_p, p_v-p *and* m_v-m), alongside the rates of profit and surplus-value and the rest of variables involved in these definitions. Note that we will be using average annual "coefficients" all throughout the paper, i.e. all variables will be expressed as magnitudes "per unit of output" and the year will be taken as the unit of time. Let us call

A the well known matrix of *technical* coefficients;

D the matrix of coefficients of *depreciation of fixed capital*, which is less common as most models consist of only circulating capital;

(with $A' = A + D$);

B the matrix of "coefficients" of *real wages*, where b_{ij} means the quantity of commodity *i* consumed (as their own means of subsistence) by the workers of industry *j* (per unit of output *j*);

and then let us ask ourselves about possible alternative ways of writing the equations of values and the rate of surplus value (in this section 2), as well as those of production prices and the rate of profit (see section 3).

i). The first and most common formulation of values—this is why this vector has the subscript *1*—is:

$$v_1' = v_1'A' + l' = l'(I-A')^{-1} \qquad (2),$$

as values are conceived of as "vertically integrated labour coefficients", being *l* the vector of *direct* labour coefficients, that become "vertically integrated" once multiplied by the Leontief inverse. There is no problem with Pasinetti's idea of vertical integration when applied to labour (see Pasinetti 1973) except one: the non integrated labour coefficients, *l*, do not reflect, as they should, the quantities of *abstract* labour which are the substance of values, but just quantities of *concrete* labour, as actually measured by the clocks at the entrance of the firms. In dealing with the relation between concrete and abstract labour, we can see again the need for beginning with macro-aggregate concepts in order to deduce correctly other variables at the microeconomic level. It is one of our main contentions that, at the aggregate level, the sum quantity of abstract labour is of the same magnitude as the sum quantity of concrete labour (in our opinion Marx believed this too), even if this is not so at any smaller level. This means that the aggregate quantity of abstract labour can and must be calculated as the total number of hours of labour *actually performed* in the year by wage workers on the whole. For example, if there were 20 million workers, each working on average 2.000 hours per year, the amount of social abstract labour would be 40.000 million hours of *human* labour. However, there is no reason to hope that one hour of concrete labour of industry *a* amounts to one hour of abstract labour at the aggregate level, or to hope that it represents the same quantity of abstract labour as one hour in industry *b*.[10]

[10] Obviously, what has been said of industries can *a fortiori* be said of firms. Note that, unlike the usual examples of Marx in *Capital*, one should not focus on individuals (persons) but on "individual

Many use the "values" defined in equation (2) for fully specifying the writing of the rate of surplus-value as it appears in (3):

$$s_1 = (L - v_1'Bx)(v_1'Bx)^{-1} \qquad (3);$$

however, as equation (2) does not represent the true values, it is obvious that equation (3), in spite of being most often attributed to Marx, should be rejected as a suitable representation of the rate of surplus-value.

ii). Any alternative to (3) needs to use values other than v_1, as is the case of (3'), that Marx would have in our opinion preferred as it makes use of other values—those expressed in straight way in market prices, m_v—for quantifying wages, which are in coherence with his general ideas about the valuation of inputs:

$$s_x = (L - m_v'Bx)(m_v'Bx)^{-1} \qquad (3').$$

(3') probably fits better with Marx's ideas since he took as given the data proportioned by the market, especially what he called *old* values—or rather values of the *old* commodities, that are "old" for the producer of his own commodities, always "new" for him[11]—when approaching his main target: the process of "valorization" or creation of *new* values. Note that being m_v the vector of so to say "market prices translated back into labour", they can be easily obtained by making use of known data: 1) the m, market prices themselves, and 2) the "monetary expression of labour time" ("melt"), e, which can in turn be deduced straightaway from m and L:

$$e = m'(I-A')xL^{-1} \qquad (4).$$

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producers", i.e. firms or even plants. Now, the staff of most firms, even if composed of the most variegated jobs, posts and positions, is much more uniform that it seems to be, and, more importantly, is confronted with a mass of means of production which are quite similar, at least inside an industry. Despite this, technical, organic and value compositions of capital clearly differ, and therefore the different concrete labour processes cannot be equated as representing the same amount of abstract labour. To be operative, if we call l the vector of concrete labour coefficients, and l_a the vector of abstract labour coefficients, we have necessarily $l_a \neq l$ and at the same time $l_a x = lx = L$. We therefore need a vector c such as every c_i, i.e. the factor of conversion of l_i in l_{ai} in industry i, is in general $\neq 1$. Obviously, we need to know l_a before calculating values as vertically integrated *abstract* labour coefficients: $l_a'(I-A')^{-1}$: see section 8 below.

[11] In analyzing changes in the value of the means of production, Marx writes: "The change of value in the case we have been considering originates not in the process of which the cotton plays the part of a means of production, and in which it therefore functions as constant capital, but in the process the cotton itself is produced. The value of a commodity is certainly determined by the quantity of labour contained in it, but this quantity is itself socially determined. If the amount of labour-time socially necessary for their production of any commodity alters (…) this reacts back on all the *old commodities* of the same type, because they are only individuals of the same species, and their value at any given time is measured by the labour socially necessary to produce them, i.e. by the labour necessary under the social conditions existing at the time" (Marx, 1867, p. 318; our emphasis).

The *melt* is just the ratio of the value added expressed in money, or *net output*, to the total labour performed. Remembering the meaning of Table 2, it is obvious that $m_v = me^{-1}$, and equation (4) represents no problem[12]. Therefore, Marx's way of formulating values would be:

$$v_x' = m_v'C + m_v'Bs_x \qquad (5),$$

where $C = A' + B$.

As we will see, v_x are not, as v_1 either, the correct values, but both of them do converge to the correct ones (see section 6).

iii). However, the correct equation of values has to use the same values for both inputs and outputs, for the reasons explained in the quote of note 11. This is probably what Marx really thought, being equation (5) not more than a proxy as he had not yet at his time the means of writing equation (6):

$$v' = v'C + v'Bs \qquad (6).$$

It is now obvious that (6) can be written in the form of eigen-equation (6'):

$$v's^{-1} = v'B' \qquad (6'),$$

where $B' = B(I\text{-}C)^{-1}$; s, the rate of surplus-value, is the reciprocal of the scalar s^{-1}, that results to be maximal eigenvalue of the transpose of matrix B'; and the vector of values, v, is the positive eigenvector associated with s^{-1}.

In the next section, when commenting the more common, symmetrical equations for the prices of production and the rate of profit, we will come back to this and explain the implications of (6) and (6'). But let us warn from now on that, instead of all appearances and contrarily to what is usually believed, these equations do not *determine* "values" together with the rate of surplus value, but just the latter and *the rates* of (relative) values.

iv). It is easily seen that there is no possible alternative of using hypothetical rates of surplus value other than s_1, s_x and s, since the resulting equation $v_2' = v_2'C + v_2'Bs_2$—or its parallel in money terms ("value prices"): $v_p' = v_p'C + v_p'Bs_2$—cannot be correct except in the case that $s_2 = s$ and $v_2 = v$.

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[12] By the same reason, we would have an equation for "production prices in value", p_v, which could be called "production values": $p_v' = m'e^{-1}C + m'e^{-1}Kr_x = m_v'C + m_v'Kr_x$.

3. The rate of profit and the prices of production

Let us examine now the several possible candidates for a correct definition of the couple "rate of profit-vector of prices of production".

i). The first of them has to be discarded by the same reasons that the first of the equations of values: because it makes use of the false values v_1 (vertically integrated *concrete* labour coefficients):

$$r_1 = (L - v_1'Bx)(v_1'Kx)^{-1} \qquad (7),$$

where K is the matrix of coefficients of stocks of constant capital (fixed and circulating) advanced. Therefore, equation (7) for the rate of profit has to be rejected, and inequality (8) can only turn into equality if $r_1 = r$ (the correct rate of profit: see below, equations 11 and 11'):

$$p_1' \neq p_1'C + p_1'Kr_1 \qquad (8).$$

ii). But before getting at the correct expressions, let us look at two different views of what a modern transcription of Marx's definition might be. The most common formulation combines the rate of profit arrived at in (7) with a set of prices of production, p_x', that valuates inputs "in value terms", "value" meaning here the false value-prices obtained from v_1:

$$p_x' = v_{p1}'C + v_{p1}'Kr_1 \qquad (9).$$

Again, we are sure that Marx would have rather preferred to define prices of production and the rate of profit as follow:

$$p_x' = m'C + m'Kr_x \qquad (9').$$

$$r_x = (L - m_v'Bx)(m_v'Kx)^{-1} \qquad (7').$$

What the couple of equations (7)-(9) has in common with (7')-(9') is that they are generally deemed to be "illogical" or "inconsistent", for critics would say that, by doing so, Marx would have forgotten to transform the value of the inputs at the same time he was in fact transforming the value of the output. However, as in the case of values, in the second interpretation of Marx's views there is no lack of transformation but rather a double one (see Guerrero, 2007). More importantly, equations (7')-(9') are not yet the correct expressions of prices of production, like equations (3')-(5) were not the correct ones for value prices either. However, the two latter couples, apart from (in our opinion) expressing correctly Marx's ideas about the valuation of inputs, show two salient additional features[13]: they *converge* towards the correct equations (10)-(10') (see section 6), and they make possible at the same time all Marx's famous *invariances* in the transformation process (section 7).

iii). Finally, in the correct formulation of the problem (symmetrical to that of equations 6 and 6' for values), the rate of profit *r* is obtained from the (eigen)-equation (10'), that is not but another way of writing the prices of production of equation (10)[14]:

$$p' = p'C + p'Kr \qquad (10)$$

$$p'r^{-1} = p'K' \qquad (10').$$

where $K' = K(I\text{-}C)^{-1}$.

4. Relative values and prices, versus absolute values and prices

The perfect symmetry between equations (6') and (10') helps us to blur the popular magic acquired by the latter when values are incorrectly defined as vertically integrated coefficients, as most often happens. Equation (10') gives us simultaneously the rate of profit and the *rates* of (production) prices, but equation (6') did not give us less the rate of surplus value and at the *rates* of values. This helps to understand that, alongside the question of the transformation of *value prices* into *production prices* (or, as we could equally say if we want to remain in the left column of Table 2, the transformation of *values* into *production values*, or "production price-values"), there is another, even more fundamental question—that of the relationship between the absolute and the relative magnitudes of both variables. It is crucial to understand that what the correct equations give us are the *rates s* and *r* on the one hand, and the *relative* values *v* (*rates* of value), and *relative* prices of production *p* (*rates* of prices) on the other hand.

But how can the idea of *absolute* values and *absolute* prices of production, *v** and *p** respectively, be developed? No theory of value can state that values and prices are fully determined unless it gives an answer to the question of the absolute magnitude of both variables: it is precisely here where one gets at the core of the theory of value. A way to elude the problem is to pose it as if it were a purely technical problem just consisting in the "normalization" or scaling of those equations. For those who want to put it in this way, it must be

[13] We deal with both in section, where we use a numerical example as illustration.
[14] Likewise, we must reject all hypothetical rate of profit other than r, like r_2, for inequality $p_2' \neq p_2'C + p_2'Kr_2$ can only be transformed into equality in the case that $r_2 = r$.

said that the theory of value requires that this scaling is not arbitrary, but deduced from its essential tenets.

Now, what the LTV affirms first of all is that labour, and only labour, creates new value; more exactly, it states that the *quantity* of labour actually spent creates a specifically determined *quantity* of value added in the economy, identical to it:

$$L = v^{*\prime}(I-A')x \qquad (11).$$

Therefore, v^* cannot be any arbitrary multiple of (6), but precisely the *single* one that is exactly determined by the LTV, i.e. the one that satisfies equation (11). And this requires scaling v as follows:

$$v^* = L[(v'(I-A')x]^{-1}v \qquad (12).$$

Only once the absolute magnitude of values is *determined* by equations (11) and (12), as well as those of prices that will be seen later, it can be said that a theoretician of value has fulfilled his goal of research. It is now obvious that, if relative values depend *only* on physical data other than labour—even if in the reality labour and the technique both go necessarily together— the magnitude of absolute values is dependent on the absolute quantity of L spent in one year.

We can turn now to production prices. Like in the case of values, the prices entering the eigenvector p are ratios or relative magnitudes, but we need to know what single and exact absolute magnitude, p^*, they must have in order for them to be compatible with the theory of value and fulfil their role in it. This can only be *determined* by means of equation (13)[15]:

$$p^* = e\cdot v^{*\prime}x(p'x)^{-1}p = v_p^{*\prime}x(p'x)^{-1}p \qquad (13),$$

which requires

$$(e\cdot v^{*\prime})x = p^{*\prime}x, \text{ or}$$

[15] There is the alternative of making $p^{*\prime} = [v_p^{*\prime}(I-A')x][p'(I-A')x]^{-1}p'$, which requires that $p^{*\prime}(I-A')x = v_p^{*\prime}(I-A')x$, and thus also $= m'(I-A')x$. In this latter case, it would not be possible to make $v_p^{*\prime}x = p^{*\prime}x$: total *gross* output would differ according to the vector chosen for evaluating the outputs, but the total *net* output would be the same not matter which vector is chosen. This choice, however, is unnecessary if we prefer to use the "approximate" equations that express Marx's posing, represented by equations (7) and (13). It is easy to see that, in this case, we have simultaneously all these invariances in spite of the change of valuation, and *both* in hours as well as in money: *gross output* ($v^{*\prime}x = p_v^{*\prime}x = m_v'x$ in hours; and $v_p^{*\prime}x = p^{*\prime}x = m'x$, in money), *net output* ($v^{*\prime}(I-A')x = p_v^{*\prime}(I-A')x = m_v'(I-A')x$, in hours; and $v_p^{*\prime}(I-A')x = p^{*\prime}(I-A')x = m'(I-A')x$, in money), *total costs* ($v^{*\prime}Cx = p_v^{*\prime}Cx = m_v'Cx$, in hours; and $v_p^{*\prime}Cx = p^{*\prime}Cx = m'Cx$, in money), *wages* ($v^{*\prime}Bx = p_v^{*\prime}Bx = m_v'Bx$, in hours; and $v_p^{*\prime}Bx = p^{*\prime}Bx = m'Bx$, in money), *profits* ($v^{*\prime}(I-C)x = p_v^{*\prime}(I-C)x = m_v'(I-C)x$, in hours; and $v_p^{*\prime}(I-C)x = p^{*\prime}(I-C)x = m'(I-C)x$, in money).

$$v_p{}^{*\prime}x = p^{*\prime}x \qquad (14).$$

It is obvious in equations (13) and (14) that, contrarily to what is commonly believed, prices of production *are not independent of values*, even if equation (10') apparently showed the opposite; in fact, what that equation showed was that the *relative* magnitude of prices of production depends solely on the physical coefficients data summarized in matrix $K' = K(I-C)^{-1} = K(I-A-D-B)^{-1}$, but it was unable to say nothing about the absolute *levels* of those prices.

In summary, the correct relationship between all variables needed for a truly complete theory of value can only be captured by the scheme of Figure 1, in which one can check that the dominant approach, by confining itself to the pointed rectangle of "Steedman's focus" (see his figure I in Steedman, 1977, p. 48), amounts to a very *partial* and biased view of the problem.

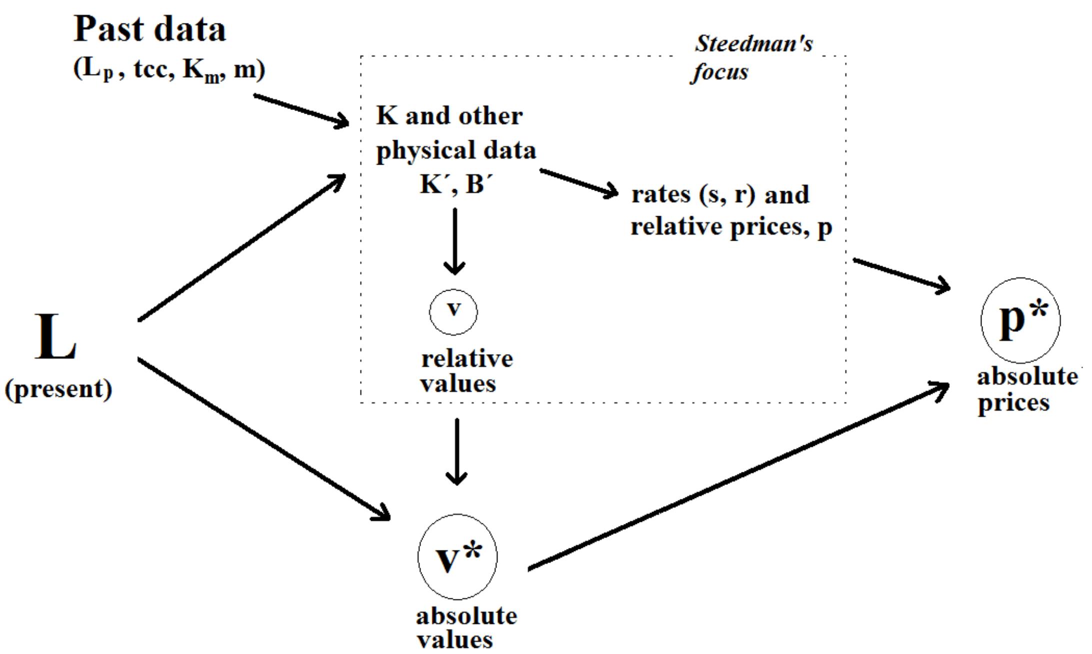

Figure 1: The complete relationship between the LTV's variables is much larger than in the usual, limited approach (with *L_p = "past" labour, or abstract labour expended in means of production; tcc = technical composition of capital; K_m = stock of capital advanced in money terms; m = vector of market prices*).

5. A numerical example

We will use now a numerical example for illustrating the kind of response undergone by values (and prices) due to changes in labour, as well as by prices (production prices) due to changes in values (value prices), while at the same time all physical coefficients are kept unaltered. Particularly, we will show through this example: 1) how values change, even if all physical coefficients do not change, as changes the quantity of labour expended in the economy (and thus changes the level of productivity, i.e. output per unit of labour); 2) how production prices too change in the same way, even if the first appearance seems to show the opposite. The second point will be illustrated as well by means of a complementary example where different countries are taken into account to show that a change in the inter-national relative level of productivity modifies the ratio between both the national vectors of value-prices, and that of the prices of production. But let us look at the general case first, where only "the economy" is present.

Suppose that an economy composed of just two industries is defined by the following four matrices of coefficients: technical coefficients (A), depreciation coefficients (D), real wage coefficients (B) and capital stock coefficients (K), apart from the identity matrix I:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad A = \begin{pmatrix} 0.09 & 0.12 \\ 0.15 & 0.08 \end{pmatrix}, \qquad D = \begin{pmatrix} 0.12 & 0.04 \\ 0.09 & 0.21 \end{pmatrix}, \qquad B = \begin{pmatrix} 0.17 & 0.12 \\ 0 & 0.1 \end{pmatrix},$$

$$K = \begin{pmatrix} 0.2 & 0.34 \\ 0.05 & 0.14 \end{pmatrix}.$$

Total hours worked are 160, distributed in both industries according to $L_s' = (80 \;\; 80)$, whereas the total output of each industry is shown by vector $x' = (40 \;\; 110)$; therefore, the vector of direct labour coefficients is $l' = (2 \;\; 0.727)$, with $l'x = 160$. As we know the vector of market prices, $m' = (6 \;\; 3)$, the "melt" (see equation 4) is easily computed and equal to $e = 1.809$. On the other hand, as matrix $B' = B(I-C)^{-1}$ amounts in the example to $B' = \begin{pmatrix} 0.426 & 0.392 \\ 0.077 & 0.199 \end{pmatrix}$ and as the rate of surplus value is the reciprocal of its dominant eigenvalue, here we have $s = 1.922$. Then, the positive left eigenvector associated with s^{-1} gives the vector of (relative) values: $v' = (0.633 \;\; 0.774)$ that, once scaled by multiplying it by $L[v'(I-A')x]^{-1}$, becomes the true vector of (absolute) values, v^*:

$$v^{*\prime} = (1.637 \;\; 2.001),$$

that is quite away from the incorrect $v_1 = (3.052 \;\; 1.712)$ generally used in this literature. Thus, in our example one unit of commodity *1* represents around $1^2/_3$ hours of labour whereas one unit of commodity *2* has a value of 2 hours. If we desire these values to be expressed in money terms, we simply multiply v^* by e to get absolute value prices expressed in, let us say, €:

$$v_p^{*\prime} = (2.963 \;\; 3.621).$$

Our initial question was: what happens to v^* if physical data (x, A, B, D, K) keep unaltered while we allow the total of hours worked be reduced, let us say, by a half, to $L_s' = (40$

40). It is easy to see that the eigenvector never changes if physical data do not change, so that v' would still be = (0.633 0.774) whereas absolute values would exactly halve to $v^{*\prime}$ = (0.819 1.001)[16]. An important thing to observe is that all this is a mental experiment, not a representation of two successive historical steps: we are just applying to the same physical matrices alternative quantities of labour[17]. This is why it does not matter to what industry we want to attribute the "fall" in the requirements of labour; and thus the result of "reducing" labour to either L_s' = (60 20), L_s' = (20 60) or (40 40)… would be the same in all cases: true values are always reduced by a half.

For the reckoning of production prices, we can repeat the same sequence: matrix K' = $K(I\text{-}C)^{-1}$ results to be in our example equal to $K' = \begin{pmatrix} 0.655 & 0.858 \\ 0.206 & 0.324 \end{pmatrix}$, being the reciprocal of the rate of profit its dominant eigenvalue, so that r = 1.062. And the positive left eigenvector associated to r^{-1} is the vector of (relative) prices of production: p' = (0.584 0.812), that, once scaled/normalized by multiplying it by $[v_p{*}'(I\text{-}A')x][p'(I\text{-}A')x]^{-1}$, carries to the vector of true (absolute) production prices:

$$p^{*\prime} = (2.679 \quad 3.724).$$

Note that these prices are expressed in money terms, and thus have to be compared to value prices (in money), not to values (in hours), in order to see that the production price of commodity one is approximately a 10% less than its value price, whereas that of the second commodity is a 3% bigger[18]. The fact of being using money prices is an important observation, since if we repeat the previous experiment we will see that by halving total labour to 80—whatever be its distribution in L_s—both the eigenvector p and the true production prices p^* remain unaffected, a result that seems to confirm the popular idea that prices are independent of values. However, this is not so: for a correct test, we must look at "production values", $p_v{}^*$ = p^*e^{-1}, and in doing so it is easy to check that they do halve (as in the case of values), lowering from (1.48 2.058) to (0.74 1.029). This example shows both that the absolute level of values is dependent on the absolute quantities of labour, and that the absolute level of prices of production is in turn dependent on the absolute level of values.

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16 Of course, value prices would remain the same $v_p{*}'$ = (2.963 3.621), since its two components move in opposite directions and with identical strength: values are reduced by one half, but e doubles to 3.619.

17 This procedure may seem strange but its rationale is clear to understand. Think of a recipe: it is obvious that the same ingredients that enter in a roast beef can be cooked in a certain quantity of labour time, let us say x hours, or in $x/2$ hours too. In practice it is difficult that this is exactly so because there is a correlation between the use of inputs other than labour and the use of (time of) labour itself (see in the consumption of energy for instance); but in keeping both strictly separate, in order to change the latter without the former, we are focusing at the indubitable fact that diverse levels of productivity of direct labour can coexist with a set of identical technical coefficients. What in practice are partially independent things we are taking here as totally independent factors, as we are just interested in conclusions that can be better seen by bearing the assumption to its extreme.

18 This is in coherence with the fact that the value composition of capital in industry 2 is higher than in industry 1, as the ratio between the vectors ($i'{*}K$) and l = (0.125 0.66). (Vector i is the unit vector.)

What the latter is not just a theoretical point but has also important practical implications can be shown by repeating the above example[19] from a new, so to say "international" perspective (see also section 9). Suppose that we look at its two logical alternatives (i.e. production of *x* with either *L* or *L*/2) as if they were two successive steps in reality, each representing a different chronological point or state in the relative position of two countries, *X* and *Y* (that we assume now to make up the entire world!): t_1 and t_2. What we are supposing is that although both countries were in identical situation in t_1, in t_2 the situation has changed but for country *Y* alone. More precisely, in t_1 each country produces *x* and requires *L* as input, thus the world produces 2*x* and uses 2*L*. By contrast, in t_2 country *X* keeps producing *x* with *L*, whereas country *Y* is now able to produce *x* making use of *L/*2 only, what in the aggregate amounts to a world production of 2*x* that requires only 1.5*L*. Thus, world productivity has increased by 33%, rising from 1 (= 2*x*/2*L*) to 1.33 (= 2*x*/1.5*L*), but this overall result is an rude average that hides the fact that the level of productivity in each country has behaved very differently: it has stagnated in country *X* and doubled in country *Y* (from 1 = *x*/*L* = 1/1, to 2 = *x*/0.5*L*). Consequently, the level of values has changed: whereas in country *X* it remains the same, in country *Y* it decreases by 50%. As a final result, both the true production prices p^* and true production values p_v^* remain unaltered in country *X*, whereas in country *Y* only the former remains unchanged, but the latter decreases by a half.[20]

[19] Before developing our numerical example, let us reflect on something that helps us to conclude that *relative* prices and values (between different countries in this case) *presuppose absolute* prices and values (in every country). Nobody would deny that the fact that in country *X* a coffee costs *the same* as a newspaper is an important practical datum, like would be the fact that the same is true in country *Y* too. However, we would ignore the most important data if we did not know *how much* those commodities cost in money in both countries, the only manner to know by the way their inter-national relative price. Nobody would deny the practical importance of knowing whether the money price of these commodities is in country *X* five times higher than in country *Y*, or the other way around.

[20] Things look differently if "world melts" are used, since in this case different national (general) levels of prices seem to appear in each country. First of all, nothing prevents us to think of a "world monetary expression" of global labour time—in fact, it would be advisable and more exact to think only in those terms, since abstract labour really encompasses the *entire* economic system. When considering the abstract case of an "economy", we have seen that the melt doubles by definition if the quantity of labour halves; however, in an international context, the single *e* becomes a set of different national e_i. If the "world melt", e_w, is substituted for the "national" ones—in country *X* e_x would keep being = 1.809 in t_2, whereas in country *Y* e_y would have doubled to = 3.619—the new numerator becomes 2*x* and the denominator 1.5*L*, so that $e_w = 1.33 \cdot e_x = 0.67 \cdot e_y$, i.e. $e_w = 2.413$. Therefore, although money prices would be *unaltered* in both countries—since where values are halved the melt has doubled (country *Y*)—and the final result seem to be the same as if nothing had changed, things are in a sense different from the world point of view. Since in t_2 $e_y = 2e_x$, it looks like if prices had doubled in country *X* as compared with those of country *Y*. The new ratio 2:1 in productivities and values results in "national levels" of prices (from the world point of view) of (3.572 4.965) and (1.786 2.483) respectively (if the share of both countries in world production were not the same, as in our example, the digits would be different but the ratio would still be 2). In order to avoid confusion, there is a need to develop this point further (see section 9) because the phrase "overall (or general) level of prices" usually refers to a single *scalar* for each country, instead of the vectors of *n* commodities we have mentioned above.

6. Convergence

When we said that neither the vertically integrated labour coefficients—the v_1 of equation (2), that will be called here v_c in order not to confuse the subscript with the first of the successive steps of the iterations we will use below—nor “Marx’s values”—the v_x of equation (5)[21]—were the true values, v^*, we advanced that both of them converged to the latter. We will show now the convergence of both vectors (points 1 and 2) and then the convergence of Marxian production prices to the correct production prices (point 3).

1. Let us see first how the iteration process of v_c runs when we replace the original vector of concrete labour, l, that was a datum, by the successive vectors l_1, l_2… that result from each previous iteration approaching us more and more to the true quantities of abstract labour. For doing this, we begin by rewriting the vector of direct concrete labour coefficients, l, as well as the v_c and s_c of equations (2) and (3) (where the subscripts refer now to the successive steps of the iteration). In the initial step (step 0) we had

$$(v_1')_0 = l'(I-A')^{-1} \qquad (2')$$
$$(s_1)_0 = [L - (v_1')_0Bx)][(v_1')_0Bx]^{-1} \qquad (3');$$

now, recalling the definition of the rate of surplus value—i.e. the ratio “surplus value/wages”, or $s = S/V$, so that $(V+S)$ can be written $(V+sV) = V(1+s)$—we can define l_1' (step 1) making use of (2’) and (3’):

$$l_1' = (v_1')_0B(1+(s_1)_0) \qquad (15),$$

and then complete the first iteration by doing

$$(v_1')_1 = l_1'(I-A')^{-1} \qquad (16)$$
$$(s_1)_1 = [L - (v_1')_1Bx)][(v_1')_1Bx]^{-1} \qquad (17).$$

Thus, successive iterations following the general formulation of the process—equations (15’), (16’) and (17’)—allow these values converge to v^*, while the rate of surplus value s_c converges to s.

$$l_i' = (v_1')_{i-1}B(1+(s_1)_{i-1}) \qquad (15')$$
$$(v_1')_i = l_i'(I-A')^{-1} \qquad (16')$$
$$(s_1)_i = [L - (v_1')_iBx)][(v_1')_iBx]^{-1} \qquad (17').$$

[21] Likewise, the same rule leads us to call s_c and s_x their respective rates of surplus value.

The numerical results of our example are put together in Table 3:

Initial data:	
	$l' = (2 \quad 0.727)$;
$(v_1')_0 = l'(I-A')^{-1}$;	$(v_1')_0 = (3.052 \quad 1.712)$;
$(s_1)_0 = [L - (v_1')_0Bx)][(v_1')_0Bx]^{-1}$;	$(s_1)_0 = 1.003$
1st iteration:	
$l_1' = v_{c0}'B(1+(s_1)_0)$;	$l_1' = (1.039 \quad 1.077)$;
$(v_1')_1 = l_1'(I-A')^{-1}$;	$(v_1')_1 = (1.907 \quad 1.946)$;
$(s_1)_1 = [L - (v_1')_1Bx)][(v_1')_1Bx]^{-1}$;	$(s_1)_1 = 1.687$
2nd iteration:	
$l_2' = (v_1')_1B(1+(s_1)_1)$;	$l_2' = (0.871 \quad 1.138)$;
$(v_1')_2 = l_2'(I-A')^{-1}$;	$(v_1')_2 = (1.706 \quad 1.987)$;
$(s_1)_2 = [L - (v_1')_2Bx)][(v_1')_2Bx]^{-1}$;	$(s_1)_2 = 1.858$
…	
8th iteration:	
$l_8' = (v_1')_7B(1+(s_1)_7)$;	$l_8' = (0.813 \quad 1.159)$;
$(v_1')_8 = l_8'(I-A')^{-1}$;	$(v_1')_8 = (1.637 \quad 2.001) = v^{*\prime} = (1.637 \quad 2.001)$;
$(s_1)_8 = [L - (v_1')_8Bx)][(v_1')_8Bx]^{-1}$;	$(s_1)_8 = 1.922 = s = 1.922$

Table 3: The process of convergence of the vertically integrated concrete labour coefficients towards the true values ($v_c \rightarrow v^*$); and of the initial rate of surplus value towards the true rate ($s_c \rightarrow s$).

2. Likewise, Table 4 shows that Marx's values v_x converge to v^* too. As in this case, due to a different definition of values, there is no need to bring l into the iterative process, the latter can be reduced to the following general formulation:

$$(s_x)_i = [L - (v_x')_{i-1}Bx][(v_x')_{i-1}Bx)^{-1} \qquad (17'')$$

$$(v_x')_i = (v_x')_{i-1}C + (v_x')_{i-1}B(s_x)_i \qquad (16'');$$

and in our example leads to the figures in Table 4:

Initial data:	
$(s_x)_0 = [L - m_v{'}Bx][m_v{'}Bx]^{-1}$;	$(s_x)_0 = 0.892$;
$(v_x{'})_0 = m_v{'}C + m{'}_vB(s_x)_0$;	$(v_x{'})_0 = (2.161\ \ 2.078)$;
1st iteration:	
$(s_x)_1 = [L - (v_x{'})_0Bx][(v_x{'})_0Bx)^{-1}$;	$(s_x)_1 = 1.421$;
$(v_x{'})_1 = (v_x{'})_1C + (v_x{'})_1B(s_x)_1$;	$(v_x{'})_1 = (1.842\ \ 2.079)$;
2nd iteration:	
$(s_x)_2 = [L - (v_x{'})_2Bx][(v_x{'})_2Bx)^{-1}$;	$(s_x)_2 = 1.679$;
$(v_x{'})_2 = (v_x{'})_2C + (v_x{'})_2B(s_x)_2$;	$(v_x{'})_2 = (1.725\ \ 2.047)$;
…	
12th iteration:	
$(s_x)_{12} = [L - (v_x{'})_{12}Bx][(v_x{'})_{12}Bx)^{-1}$;	$(s_x)_{12} = 1.922 \quad = \quad s \quad = 1.922$;
$(v_x{'})_{12} = (v_x{'})_{12}C + (v_x{'})_{12}B(s_x)_{12}$;	$(v_x{'})_{12} = (1.637\ \ 2.001) = v^{*\prime} = (1.637\ \ 2.001)$;

Table 4: The process of convergence of Marx's values, v_x, towards the true values, v^*; and of the Marxian rate of surplus value, s_x, towards s.

3. Lastly, we will focus on the convergence of Marxian production prices towards the true production prices. The iterative process, whose general formulation is now:

$$(p_x{'})_i = (p_x{'})_{i-1}(C + K(r_x)_{i-1}) \qquad (18)$$

$$(p'_{vx}{'})_i = (p_x{'})_i e^{\prime -1} \qquad (19)$$

$$(r_x)_i = [L - (p'_{vx}{'})_iBx][(p'_{vx}{'})_iKx)^{-1} \qquad (20),$$

leads us to p^* once the melt is “adjusted” from e to e'[22] and the corresponding production values, p'_{vx}, redefined as in equation (19), are properly used in the formulation of the rate of profit. When we apply these equations to our numerical example, the iteration runs as follows:

[22] Note that we have been using up to now $e = m{'}(I-A{'})xL^{-1}$, whereas e_p is $= p^{*\prime}(I-A{'})xL^{-1}$, i.e. the monetary expression of labour time when true prices of production are used.

Initial data:	
$(r_x)_0 = [L - m_v'Bx][m_v'Kx]^{-1}$;	$(r_x)_0 = 0.421$;
$(p_x')_0 = m'(C + K(r_x)_0)$;	$(p_x')_0 = (3.568 \;\; 3.884)$;
1st iteration:	
$(p_x')_1 = (p_x')_0(C + K(r_x)_0)$;	$(p_x')_1 = (2.67 \;\; 3.253)$;
$(p'_{vx}')_1 = (p_x')_1 e^{-1}$;	$(p'_{vx})_1 = (1.46 \;\; 1.779)$;
$(r_x)_1 = [L - (p'_{vx}')_1 Bx][(p'_{vx}')_1 Kx)^{-1}$;	$(r_x)_1 = 1.144$;
2nd iteration:	
$(p_x')_2 = (p_x')_1(C + K(r_x)_1)$;	$(p_x')_2 = (2.592 \;\; 3.576)$;
$(p'_{vx}')_2 = (p_x')_2 e^{-1}$;	$(p_{vx})_2 = (1.417 \;\; 1.955)$;
$(r_x)_2 = [L - (p'_{vx}')_2 Bx][(p'_{vx}')_2 Kx)^{-1}$;	$(r_x)_2 = 1.12$;
…	
13th iteration:	
$(p_x')_{13} = (p_x')_{12}(C + K(r_x)_{12})$;	$(p_x')_{13} = (2.679 \;\; 3.724) \quad = \quad p^{*\prime}$
$(p'_{vx}')_{13} = (p'_x')_{13} e^{-1}$;	$(p_{vx})_{13} = (1.465 \;\; 2.036)$;
$(r_x)_{13} = [L - (p'_{vx}')_{13} Bx][(p'_{vx}')_{13} Kx)^{-1}$;	$(r_x)_{13} = 1.062 \quad = \quad r \;\; = 1.062$;

Table 5: The process of convergence of the Marxian rate of profit, r_x, towards r, and of Marx's production prices, p_x, towards the correct production prices, p^*.

7. Invariances

It is commonly attributed to Marx the oblivion of transforming inputs at the same time as outputs, or the inability to do so. We have suggested that it is possible that he were thinking of inputs, both *before and after* transformation, as being evaluated not at untransformed values, as is commonly believed, but at values already transformed *twice*, i.e. at values being the direct translation in hours of given actual market prices—i.e. values obtained after transforming labour values into production values (first transformation), and then the latter into the values expressed as market prices (Second transformation). If this is so, as suggested by Guerrero (2007), *"Marx's" variables*, that bear a subscript *x*, would be defined and quantified as in Table 6, where values are v_x; value-prices, v_{px}; production prices, p_x; production values, p_{vx}; and the rate of surplus value and the rate of profit, s_x and r_x respectively. Physical data (the matrices and *x*) and market prices and values, *m* and m_v, are the same as assumed in the rest of definitions studied in this paper. Table 6 shows that—contrarily to what happens *at the individual level*, where just costs coincide, but not profits nor value added nor output—the magnitude of overall output, as well as that of its components, are the same *at the aggregate level*, no matter whether they are reckoned before or after the process of transformation, and this happens in both money and labour terms.

I. In money terms	**Value price** per unit	**Value price** Total	**Production price** Total	**Production price** per unit
Components:				
material cost	$m'A' =$ **(1.98 1.83)** =	$m'A'x =$ **280.5 €** =	$m'A'x =$ **280.5 €**	$mA' =$ **(1.98 1.83)** =
value added	$v_{px}' - m'A' =$ (1.93 1.93) ≠	$(v_{px}' - m'A')x=$ **289.5 €** =	$(p_x'-m'A')x =$ **289.5 €**	$p_x' - m'A' =$ (1.588 2.054) ≠
· wage cost	$m'B =$ **(1.02 1.02) =**	$m'Bx =$ **153 €** =	$m'Bx =$ **153 €**	$m'B =$ **(1.02 1.02)** =
· profit	$m'Bs_x =$ (0.91 0.91) ≠	$m'Bs_xx =$ **136.5 €** =	$m'Kr_xx =$ **136.5 €**	$m'Kr_x =$ (0.568 1.034) ≠
Output	$v_{px}' =$ (3.91 3.76) ≠	$v_{px}'x =$ **570 €** =	$p_x'x =$ **570 €**	$p_x' =$ (3.568 3.884) ≠
		***		
II. In hours	**Value** per unit	**Value** Total	**Production value** Total	**Production value** per unit
Components:				
material cost	$m_v'A' =$ **(1.094 1.011) =**	$m_v'A'x =$ **155.026 H.** =	$p_{vx}'A'x =$ **155.026 H.**	$p_{vx}'A' =$ **(1.094 1.011)** =
value added	$v_x' - m_vA' =$ (1.067 1.067) ≠	$(v_x' - m'A')x =$ **160 H.** =	$(p_{vx}' - m_v'A')x=$ **160 H.**	$(p_{vx}' - m_v'A') =$ (0.877 1.135)
· wage cost	$m_v'B =$ **(0.564 0.564) =**	$m_v'Bx =$ **84.56 H.** =	$p_{vx}'Bx =$ **84.56 H.**	$p_{vx}'B =$ **(0.564 0.564)**
· surplus value	$m_v'Bs_x =$ (0.503 0.503) ≠	$m_v'Bs_xx =$ **75.44 H.** =	$p_{vx}'Kr_xx =$ **75.44 H.**	$p_{vx}'Kr_x =$ (0.314 0.572)
Output	$v_x' =$ (2.161 2.078) ≠	$v'x =$ **315.026 H.** =	$p_{vx}'x =$ **315.026 H.**	$p_{vx}' =$ (1.972 2.147)

Table 6: When values and production-values, as well as value prices and production prices are defined *à la Marx*, all of his invariances hold in Transformation, and there is just one single rate of profit.

By contrast, Table 7 shows that things are different when the correct values and prices are used. As it is well known, in this case only one invariance can be maintained—the overall outputs in our example—and it can be shown that this happens when measured both in money and in hours of value. However, what can seem more surprising to some critics of the LTV is that there is *a single rate of profit*, since *r*, the reciprocal of the maximal eigenvalue of *K'*, is exactly equalled by the rate of profit “in value” of equation (21)[23]:

$$r_L = (L - p_v{}^{*\prime}Bx)(p_v{}^{*\prime}Kx)^{-1} \qquad (21),$$

which in our example is $r_L = r = 1.062$. In fact, the latter conclusion adds to the idea that there is really no rationale for distinguishing between a rate of profit in value and a rate of profit in price: as prices are the expressions of values, the ratio “surplus value-capital advanced” has to be the same no matter whether the comparison is made in quantities of money or in quantities of labour; and the same happens with the rate of surplus value and other variables of the LTV.

[23] Note that we have passed from p^* to $p'_v{}^*$ by using again $e_p = p^{*\prime}(I-A')xL^{-1}$, so that $p'_v{}^* = p^*e_p^{-1}$.

I. In money terms	**Value price** per unit	Total	**Production price** Total	per unit
Components:				
material cost	$v_p{*}'A' =$ (1.491 1.524) ≠	$v_p{*}'A'x =$ 227.293 € ≠	$p{*}'A'x =$ 224.195 €	$p{*}'A' =$ (1.456 1.509) ≠
value added	$v_p{*}'(I\text{-}A') =$ (1.472 2.097) ≠	$v_p{*}'(I\text{-}A')x =$ 289.5 € ≠	$p{*}'(I\text{-}A')x =$ 292.599 €	$p{*}'(I\text{-}A') =$ (1.222 2.215) ≠
· wage cost	$v_p{*}'B =$ (0.504 0.718) ≠	$v_p{*}'Bx =$ 99.084 € ≠	$p{*}'Bx =$ 94.539 €	$p{*}'B =$ (0.455 0.694) ≠
· profit	$v_p{*}'Bs =$ (0.968 1.379) ≠	$v_p{*}'Bsx =$ 190.416 € ≠	$p{*}'Krx =$ 198.059 €	$p{*}'Kr =$ (0.767 1.522) ≠
Output	$v_p{*}' =$ (2.963 3.621) ≠	$v_p{*}'x =$ **516.793 €** =	$p{*}'x =$ **516.793 €**	$p{*}' =$ (2.679 3.724) ≠
		***		
II. In hours	**Value** per unit	Total	**Production value** Total	per unit
Components:				
material cost	$v{*}'A' =$ (0.824 0.842) ≠	$v{*}'A'x =$ 125.62 H. ≠	$p_v{*}'A'x =$ 123.907 H.	$p_v{*}'A' =$ (0.805 0.834) ≠
value added	$v{*}'(I\text{-}A') =$ (0.813 1.159) ≠	$v{*}'(I\text{-}A')x =$ 160 H. ≠	$p_v{*}'(I\text{-}A')x =$ 161.712 H.	$p_v{*}'(I\text{-}A') =$ (0.676 1.224) ≠
· wage cost	$v{*}'B =$ (0.278 0.397) ≠	$v{*}'Bx =$ 54.761 H. ≠	$p_v{*}'Bx =$ 52.25 H.	$p_v{*}'B =$ (0.252 0.383) ≠
· surplus value	$v{*}'Bs =$ (0.535 0.762) ≠	$v{*}'Bsx =$ 105.239 H. ≠	$p_v{*}'Krx =$ 109.463 H.	$p_v{*}'Kr =$ (0.424 0.841) ≠
Output	$v{*}' =$ (1.637 2.001) ≠	$v{*}'x =$ **285.62 H.** =	$p_v{*}'x =$ **285.62 H.**	$p_v{*}' =$ (1.48 2.058) ≠

Table 7: When values, production-values, value-prices and production prices are correctly defined, just one invariance holds in Transformation (in the general case), but there is still *a single* rate of profit.

8. Concrete labour and abstract labour

We have said that what is commonly taken as the vector of direct labour, l, is not a magnitude of abstract labour but of concrete labour instead. So we need to convert l into abstract labour, l_a, if we want to continue thinking of true values. Now, once clarified in this paper the correct definition of values and remembering the relation between values and the rate of surplus value, we can write the vector of abstract direct labour as:

$$l_a' = v{*}'B(1+s) \qquad (22).$$

In our example, we see that l_a = (0.813 1.159)[24], and it can be easily seen that:

$$l_a'x = l'x = 160.$$

[24] The same is true if we start with the definitions of v_c and s_c, or from that of v_x and s_x, and iterate from them. In both cases we arrive to (0.813 1.159) too, at steps 7 and 9 of the iteration respectively.

If we compare vectors l_a and l, we obtain what might be called “coefficients of reduction” of the different concrete labours of every industry to quantities of homogeneous, abstract human labour. In our example, as l was = (2 0.727), and the *ratio* between l_a and l is not (1 1) but results to be = (0.407 1.593), we can deduce from this data that in the first industry of our example one hour of concrete labour just forms 24.4 minutes of abstract labour, whereas 1 hour of concrete labour in industry 2 represents almost 4 times more abstract labour, exactly 95.6 minutes. This shows that going from concrete labour to abstract labour looks like a “redistribution” of total labour among industries, a result that can be seen as well in terms of the *total* number of hours worked in every industry instead of in terms of labour per unit of output. If we weight the hours of total concrete labour, Ls = (80 80), by the above ratio, we get at L_{sa} = (32.533 127.467), that means that 80 hours of concrete labour have a very different potential of generating abstract labour in the two industries, as if there had been worked 32.5 and 127.5 hours of labour respectively.

However, it should be noted that the transition from concrete labour to abstract labour can be interpreted in a *double* way, as there are two different magnitudes of value that need be taken into consideration: values and production values. So to say, the process of (practical) abstraction of labour has to be measured differently according to the level of (theoretical) abstraction we are using in each step of analysis—either the level of value prices or that of production prices. If we are at the latter level, 1 hour of concrete labour does not need to represents the same quantity of abstract labour as compared to that represented in value prices. To this purpose, we should use equation (18’) instead of (18):

$$l_{a2}' = p'_v{}^{*\prime}(B+rK) \qquad (22'),$$

where $p'_v{}^*$ is the vector of absolute “production values” (equal to $p^*e_p^{-1}$). In our example l_{a2}' = (0.668 1.211), that differs from l_a' = (0.813 1.159), but:

$$l_{a2}^{*\prime}x = l_a'x = l'x = 160.$$

In this case, the ratio between l_{a2} and l is even greater than in the first case—(0.334 1.666)—meaning that 1 hour of concrete labour in industry 1 amounts to just 20.054 minutes of abstract labour present in its unit price of production, whereas in industry 2 it amounts to 99.946 minutes (almost five times more). Alternatively, we can say that total concrete labours of (80 80) hours must be converted into (26.738 133.262) hours of abstract labour.

Finally, an alternative but completely equivalent way to arrive at the vector of abstract labour is to multiply the quantities of concrete labour in each industry by the relative *value-wage* of that industry (i.e. its particular wage as compared to the average wage in the economy). Note that it is *not* actual market wages what we take as reference—as Shaikh (1984), Ochoa (1989), Guerrero (2000) and others have done in their empirical work—but wages measured in pure labour-value terms. Let us see. As *B* is the matrix of coefficients of *real* wage, $w_u' = v^{*\prime}B$ would be the vector of coefficients of wages measured *in value* ("per unit of output" in every industry); w_{pc} the vector of value-wages "per capita" in every industry (i.e. w_u divided by l); and $w = w_u'x/l'x$ the average wage per capita for the overall economy. Therefore, the ratio between w_{pc}/w gives us the vector w_r of *relative* wages of all industries. It is easy to see that in our example w_r = (0.407 1.593), which coincides with the ratio between abstract and concrete labour calculated previously.

9. From the General level of values to the General level of prices

It is well known that the most common interpretation of the so called "equation of money" (or "equation of exchange"), the identity $PQ \equiv MV$, is given by the "quantity theory of money", to which—put it in a simple way—the general level of prices, *P*, would be proportional to the quantity of money, *M*, if both components of the ratio (*V/Q*), i.e. the velocity of money, *V*, and the real volume of output, *Q*, were constant. However, in coherence with his LTV, Marx challenged this interpretation, supporting instead the view that the identity should be interpreted the other way around (see equation 23); put it in the same simplified form, it could be said that the quantity of money required by the economy depends on the general level of prices, so that it would be proportional to the latter if *Q* and *V* were supposed constant:

$$M = (Q/V) \cdot P \qquad (23).$$

If this is so, one should show how *P* is determined in order to complete the explanation, which we intend to do in what follows. The "general level of prices" is a weighted[25] average of the absolute or money prices of all commodities, but this idea can be referred to whichever of the three types of prices we ha studied in this paper: *m*, *p* or v_p. Thus, we should distinguish between the three different scalars P_m, P_p and P_{vp}, defined as:

$$P_m = \sum_i m_i \cdot z^m_i = m'z^m \qquad (24)$$

[25] We should be well aware of a the fact that the absolute magnitude of *P* depends crucially on the exact definition given of the *physical units* of all commodities in the system, as well as on the *industrial structure* of production determining the weights of each price in the general level. Suppose that all physical units are identically defined in two countries whatever, *X* and *Y*: it is obvious that even then, and assuming as well that both countries share the same vectors of unit values and of unit prices, neither the national scalars *P* nor *V* (see below) need to be the same in both countries, so that in general $P^X \neq P^Y$ and $V^X \neq V^Y$. This is why authors and institutions think it is wiser to use index numbers for *P* rather than absolute levels; however, those indexes are also dependent on the different rhythm of change in the structure of production and thus are not completely reliable either.

$$P_p = \sum_i p^*_i \cdot z^p_i = p^{*\prime} z^p \qquad (24')$$

$$P_{vp} = \sum_i v_p{}^*_i \cdot z^v_i = v_p{}^{*\prime} z^v \qquad (24''),$$

where the z_i are the weights of each price, given by the fractions $z^m_i = m_i x_i (m'x)^{-1}$, $z^p_i = p_i x_i (p'x)^{-1}$, $z^v_i = p_{vi} x_i (p_v'x)^{-1}$; and z^m, z^p and z^v are the vectors formed by the elements z^m_i, z^p_i and z^v_i respectively. We have seen that, according to the LTV, the absolute prices of production had to be defined as:

$$p^{*\prime} = p'(v_p{}^{*\prime} x)(p'x)^{-1} \qquad (13).$$

The most common way, however, to look at the normalization of vector p' is not this one, but a result of using the “equation of money” as interpreted by the defenders of the quantity theory of money. Following the steps long ago criticized by Patinkin, this alternative would determine the absolute prices $p^{**\prime}$, beginning from relative prices p', by making use of equation (25), that is defined by means of the general level of prices:

$$p^{**\prime} = p'(P_p Q)(p'x)^{-1} \qquad (25)$$

Now, if $p^{**\prime}$ is to be the correct vector, it should be = $p^{*\prime}$, and this requires that $P_p Q = v_p{}^{*\prime} x$. It is easy to see in what conditions this equality can take place: as Q is the scalar for the quantity of output in real terms, it must be equal to the money value of total output divided by the general level of prices ($Q = Y^R = Y/P$), so that

$$Q = (p'x)(p'z^p)^{-1} \qquad (26)$$

and $P_p Q = (p^{*\prime} z^p) \cdot (p'x)(p'z^p)^{-1}$. As the latter has to be = $v_p{}^{*\prime} x$, we have

$$p^{*\prime} z^p = [v_p{}^{*\prime} x (p'x)^{-1}] p' z^p,$$

which is only possible if

$$p^{*\prime} = [v_p{}^{*\prime} x (p'x)^{-1}]\, p',$$

as required by the LTV.

We have proved in this way that any normalizations of prices that make no use of value prices—the money expression of labour-values—can not be correct[26]. Prices are thus shown to

[26] We can come back to our numerical example to check this. If defined as $P = p^{*\prime} z^p$, we get $P = 3.507$; $PQ = 516.793 = vp^{*\prime} x$; and $p^* = (2.679 \quad 3.724)$, as before. By contrast, if we did $P = p' z^p$ (as all normalizations have to begin with non normalized prices), the same data would give $P = 0.765$; $PQ =$

be dependent on values. By according this role to values—absolute values that are therefore *cardinal*—as regulators of national and international levels of prices, it is made possible to avoid as well the unjustified cut between microeconomics and macroeconomics, repeatedly denounced at least since the beginning of the famous *Patinkin debate*[27].

An alternative way to see this is the following. Let us call "general level of values", V, the labour replicae of equations (24), (24') and (24''), of which we have three:

$$V_m = \sum_i v_{mi} \cdot z^m{}_i = v_m{}' z^m \qquad (27)$$

$$V_p = \sum_i p_v{}^*{}_i \cdot z^p{}_i = p_v{}^{*\prime} z^p \qquad (27')$$

$$V_{vp} = \sum_i v^*{}_i \cdot z^v{}_i = v^{*\prime} z^v \qquad (27'').$$

It we choose the second and write $V_p = (e^{-1}p^{*\prime})z^p$, then:

$$P_p = eV_p = ep_v{}^{*\prime} z^p = e[p'(v^{*\prime}x)(p'x)^{-1}]z^p,$$

that is equal to:

$$P_p = e[p'z^p(p'x)^{-1}](v^{*\prime}x) =$$
$$= eQ^{-1}(v^{*\prime}x) = eQ^{-1}W \qquad (28)^{28}.$$

where W is the value of total output. Similarly, we would have

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112.655; and $p^{**\prime} = (0.584 \;\; 0.812) = p' \neq p^* = (2.679 \;\; 3.724)$. It is obvious that any prices different to p^* give a result that differs from the true one.

[27] Patinkin was right in denouncing "the classical dichotomy between the real and monetary sectors", i.e. the habit of "determining relative prices in the real part of the model, and absolute prices through the money equation" (Patinkin, 1949, pp. 2, 21). He was well aware that "there is no monetary equation that we can use to remove this indeterminacy of absolute prices" (ibid., p. 21), but ignored that the LTV does offer an explanation that lacks in neoclassical economics. When he writes that "the only way to have the system determine absolute prices is to have them appear in the real sector of the economy too", he is showing his unawareness that there is in fact no "real sector" that can be severed from a "monetary sector": in a capitalist economy, money represents labour, quantities of abstract labour, and this is really why "the real sector does not provide enough information to complete this task; at most it can determine all but one of the prices as functions of the remaining one" (ibid., p. 2). Put in other terms, the inability of the "relative" approach to the theory of prices should not be transferred to the LTV's "absolute" approach, once it has been shown that labour can overcome the "classical dichotomy". Finally, it is not true that "the only way out" of the problem is "to recognize that prices are determined in a truly general-equilibrium fashion, by both sectors simultaneously" (ibidem). What is needed for this purpose is to allow the LTV to explain, as made in this paper, how absolute prices are determined at both the individual or microeconomic (industry) level, and the aggregate or macroeconomic level. Once understood that labour is the only factor creating new value, and that values have to be necessarily expressed in money terms (as absolute prices), the false dichotomy between a real sector and a monetary sector disappears, and theoretical unity can be restored. After having seen that the LTV is the only theory of value that can aspire to be *complete*, beyond relative prices, we have seen that it is also the only one that can offer a *unified* vision of the capitalist economy, where production and money really belong to the same world and prices have to be understood as historical expressions of labour relations.

[28] Note that we have, from (4), $P_pQ = eW = e \cdot v^{*\prime}x = v_p{}^{*\prime}x$, that is the known requirement for $p^{**\prime} = p^{*\prime}$.

$$V_p = e^{-1}P_p = Q^{-1}W \qquad (29)$$

and it is clear, by comparing equations (28) and (29), that if we divide L by 2, such that $v^{*\prime}$ and W are also divided by 2, then V_p results halved too. On the contrary, P_p remains unaltered since a division of L by 2 makes e double, thus compensating the decrease in V_p.

Therefore, computing values as made in this paper is an exact manner of quantifying the overall and weighted effect of the innumerable and simultaneous changes in productivity in all industries and countries in the world. These changes can be thought of as explaining the *basic, long run paths of the national general levels of prices*, and thus the evolution of nominal and real exchange rates between national currencies, without having a need to look simultaneously at the monetary factors; these factors, including changes in the quantitative relationship between different forms of money in every country (in particular, the volume of credit *vis à vis* the metallic base of the bank system), must enter the scene at a second moment only, as they are just behind the *short run deviations* from the long run path.

10. Conclusions

We have reached in this paper a number of important conclusions, the most important of which can be listed here:

1. Labour values are generally incorrectly defined as vertically integrated direct *concrete* labour coefficients. Of course, those "values" have nothing to do with prices of production. However, once abstract labour is correctly defined and calculated, it can be checked that values are vertically integrated direct *abstract* labour coefficients. At the same time, this allows us to know how to reduce quantities of concrete labour to quantities of abstract labour, which is made here at the industry level.

2. *Relative* values can be calculated in a parallel fashion with production prices: as the only positive left eigenvectors of two different input-output matrices deduced from physical data and the real wage. The rate of surplus value and the rate of profit are the reciprocal of the eigenvalues of those matrices (and there is one single rate of profit, not two as is generally believed). However, *absolute* values and prices of production cannot be known from those data only. With the same physical data, the level of the vector of values is proportional to the overall quantity of labour performed in the economy, and the level of the vector of production values—the labour counterpart of production prices—is proportional to the magnitude of labour values.

3. As the general level of prices (a scalar) is a weighted average of the vector of prices it is also dependent on the quantity of labour and value. At the same time, as money is the

necessary expression of labour, the LTV offers a correct *causal explanation* of the identity called the equation of money, as seen in the fact that in order to obtain the absolute level of prices no scaling of relative prices can be offered by any theory of money other than that shown in this paper, that starts from the *equality of value prices and production prices* at the aggregate level.

4. It is not true that Marx forgot to transform the inputs in his Transformation procedure. Simply, as he focused on the process of creation of new values he took as given the values of the inputs that are “old” in a logical, not chronological sense. This is why he always evaluated the inputs at *values proportional to market prices*, in defining his values as well as his prices of production. Defined in this manner, these values and production prices are not exactly the correct ones but, as they *converge* to them, they can be interpreted as an excellent first approach to them. Moreover, if they are used to illustrate the Transformation procedure, all *Marx’s invariances rule*.

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