Comparison of Optimal Portfolios Selected by Multicriterial Model Using Absolute and Relative Criteria Values

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ABSTRACT:

In this paper we select an optimal portfolio on the Croatian capital market by using the multicriterial programming. In accordance with the modern portfolio theory maximisation of returns at minimal risk should be the investment goal of any successful investor. However, contrary to the expectations of the modern portfolio theory, the tests carried out on a number of financial markets reveal the existence of other indicators important in portfolio selection. Considering the importance of variables other than return and risk, selection of the optimal portfolio becomes a multicriterial problem which should be solved by using the appropriate techniques.

In order to select an optimal portfolio, absolute values of criteria, like return, risk, price to earning value ratio (P/E), price to book value ratio (P/B) and price to sale value ratio (P/S) are included in our multicriterial model. However the problem might occur as the mean values of some criteria are significantly different for different sectors and because financial managers emphasize that comparison of the same criteria for different sectors could lead us to wrong conclusions. In the second part of the paper, relative values of previously stated criteria (in relation to mean value of sector) are included in model for selecting optimal portfolio.

Furthermore, the paper shows that if relative values of criteria are included in multicriterial model for selecting optimal portfolio, return in subsequent period is considerably higher than if absolute values of the same criteria were used.

RESUMEN:

En el presente artículo seleccionamos la cartera de inversiones óptima dentro del mercado de capitales de Croacia utilizando una programación multicriterio. De acuerdo con la teoría moderna de gestión de carteras, la maximización del rendimiento con riesgo mínimo debería ser el objetivo de todo buen inversor. Sin embargo, contrariamente a las expectativas generadas por la teoría moderna de gestión de carteras, los estudios realizados en varios mercados financieros revelan la existencia de otros indicadores importantes para elegir carteras de inversión. Considerando la importancia de otras variables además del rendimiento y el riesgo, la elección de la cartera idónea se convierte en un problema multicriterio que se resuelve utilizando técnicas apropiadas.

Para elegir la cartera de inversión óptima, incluimos en nuestro modelo multicriterio los valores absolutos de criterio, como el rendimiento, el riesgo, la relación precio-beneficios (por sus siglas en inglés P/E), la relación precio-valor contable (P/B) y la relación precio-ventas (P/S). Sin embargo, el problema aparece cuando los valores medios de algunos criterios son significativamente diferentes para diversos sectores, debido a que los gestores financieros hacen hincapié en el hecho de que la comparación de los mismos criterios aplicados a distintos sectores nos puede dar resultados erróneos. En la segunda parte de nuestro trabajo, los valores relativos de los criterios anteriormente mencionados (en relación con el valor medio del sector) se incluyen en el modelo para elegir la mejor cartera de inversión.

Finalmente, demostramos que si incluimos los valores relativos de los criterios aplicados en nuestro modelo multicriterio a la hora de elegir la mejor cartera de inversión, el rendimiento en el siguiente período es considerablemente mayor que si se utilizaran los valores absolutos de dichos criterios.

1. INTRODUCTION

In 1952 H. M. Markowitz developed the first model for portfolio optimization and with that model he placed foundation of the modern portfolio theory. His model is based upon only two criteria: a return and a risk. The risk is measured by the variance of returns' distribution. Markowitz shows how to calculate portfolio which has the highest expected return for a given level of risk, or the lowest risk for a given level of expected return (so-called efficient portfolio). The problem of portfolio selection, according to this theory, is a simple problem of quadratic programming which consists in minimizing risk while keeping in mind an expected return which should be guaranteed.

The importance of Markowiz's work is reflected in the fact that he won the Nobel Prize for economics in 1990. However, parallel to introducing the Markowiz's model in common usage its limitations and drawbacks were being noticed. The assumptions of the Markowitz model for portfolio optimization are the following:

- utility function which presents the investor's preferences is a quadratic function
- the returns have normal distribution

These assumptions were the starting point of many critics of this model. The majority of the empirical tests on the capital markets resulted in assymetrical and (or) leptokurtic distribution[1]. At such distributions variance is not an adequate risk measure. Having recognized the drawbacks of variance as risk measure new models for the selection of optimal portfolio which use alternative measure [2] have been developed.

However, contrary to the expectations of the modern portfolio theory, the tests carried out on a number of financial markets have revealed the existence of other indicators, beside return and risk, important in portfolio selection. Considering the importance of variables other than return and risk, selection of the optimal portfolio becomes a multicriterial problem which should be solved by using the appropriate techniques. The multi-criteria nature of the portfolio selection is well presented in the paper of Khoury et al. still 1993, and many multi-criteria methods have already applied in portfolio selection[3].

There are a vast number of criteria that can be taken into consideration in portfolio selection and they are usually classified into two groups: accounting criteria and those based on market values. The accounting criteria are obtained analyzing audit reports, income statement, quarterly balance sheets, dividend records, sales records, etc. There are a large number of them like profitability indicators, liquidity and solvency indicators and indicators of financial structure of the company. They are used by the analysts (or managers) to give a synthesized and clear idea about the firm's financial situation. Second criteria are market criteria and they contain all the information used by the stock analysts to appreciate a stock's performance. The criteria used at this level are the mean return, total risk (variance), systematic risk (beta), the size measured by the stock capitalization, the PER (price earning ratio), stock liquidity and others. The use of the one criteria or the other depends on the manager's attitude and objectives.

Although all these criteria are important in selecting optimal portfolio the question arises whether different sector stocks are comparable on all these criteria.

The problem might occur as the mean values of some criteria are significantly different for different sectors and because financial managers emphasize that comparison of the same criteria for stocks of different sectors could lead us to wrong conclusion.

To solve this problem we introduce relative criteria value defined as ratio of absolute value criteria for certain stock and mean criteria value of the corresponding sector. Furthermore we select optimal portfolio using absolute and relative criteria values respectively and then we compare optimal portfolios selected by absolute and relative criteria values.

This paper is organized as follows: following this introduction, Section 2 provides the research approach. In Section 3, we present the multi-criterion procedure. Section 4 presents the application to Croatian capital market. Section 5 summarizes the paper and indicates the possible directions for further research.

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[1]Cloquette, J.F., Gerard, M., Hadhri, M., (1995)

[2]Rockafaller, R. T. I Uryasev, S., (2000); Konno, H., Waki, H., Yuuki A., (2002)

2. RESEARCH APPROACH

The problem addressed by this paper refers to the comparability of stocks of different sectors on particular criteria. Namely, stocks of different sectors cannot be compared on those criteria the mean value of which is significantly different in different sectors. For such criteria we introduce relative criteria value defined as ratio of absolute value criteria for certain stock and mean criteria value of corresponding sector, i.e.

Relative criteria value for certain stock = absolute value criteria for certain stock / mean criteria value of corresponding sector

Aiming at proving that stock evaluation based on the relative values of "problematic" criteria gives better results that stock evaluation based on the absolute values of those criteria we select optimal portfolio by multicriterial model using absolute and relative criteria values respectively and then we compare optimal portfolios selected by absolute and relative criteria values.
The comparison of the obtained portfolios will be carried out by taking returns of pre-selected stocks in subsequent period and calculating return of each optimal portfolio in subsequent period.
The selected method we will be applied on Zagreb Stock Exchange (ZSE) as a real case. Zagreb Stock Exchange (ZSE) is a major stock market in Croatia and its market value is more than sixsty billion dollars.

3. THE MULTI-CRITERION PROCEDURE

The multi-criterial method applied in this paper is based on the PROMETHEE approach. In accordance with the PROMETHEE method each alternative *P* (in our case portfolios) are evaluated with two flows. Positive flow $\Phi^+(P)$ indicates how much the alternative is better than the others (on all criteria). Accordingly, the higher the $\Phi^+(P)$ the better the alternative. Negative flow $\Phi^-(P)$ indicates how much better than *P* the other alternatives are, i.e. how much *P* is dominated by the others. Accordingly, the lesser $\Phi^-(P)$ the better the alternative.

PROMETHEE II calculates the net flow Φ as the difference between the two flows, i.e.

$$\Phi(P) = \Phi^+(P) - \Phi^-(P),$$

so the higher the net flow $\Phi(P)$ the better the alternative. Positive and negative flows are calculated by comparison of pairs of all alternatives.

Since the overall portfolios that can be made up from a set of pre-selected shares are infinite, it is impossible to compare all of the pairs of portfolios. Therefore, Khoury and Martel (1990) as well as Zmitri (1998) suggest a different procedure which evaluates each alternative (or rather its positive and negative flow) of the calculation by comparison with two fictions portfolios: one ideal ($\overline{P}$) and the other anti-ideal ($\underline{P}$). The positive flow $\Phi^+(P)$ is then obtained by comparison with anti-ideal, where the higher $\Phi^+(P)$ the better the alternative (it could be said the more distant from the anti-ideal it is). Accordingly, the closer to the ideal the better the alternative or the lesser the $\Phi^-(P)$ the better the alternative. According to the PROMETHEE II method the portfolio is better if it has a higher net flow Φ .

For criteria C_j to maximize we'll have $C_j(\overline{P}) = \max_i C_j(s_i)$, where $S = \{s_1, s_2, ..., s_N\}$ is the set of *N* pre-selected shares. In the same way $C_j(\underline{P}) = \min_i C_j(s_i)$. The set of possible solutions is the set of portfolios which can be formed from pre-selected shares. For any possible portfolio *P* has to be $\Phi(\underline{P}) \le \Phi(P) \le \Phi(\overline{P})$.

Positive and negative flows are calculated separately for each particular criteria, i.e. for each criteria C_j $(j = 1,2,...,n)$ we have to calculate $\Phi_j^+(P)$ i $\Phi_j^-(P)$, and the net flow is obtained as a weighted sum of the difference between those flows, i.e.

$$\Phi(P) = \sum_{j=1}^{n} w_j \left(\Phi_j^+(P) - \Phi_j^-(P) \right)$$

where w_j are weights of the criteria obtained in agreement with the decision maker (or some of the methods, e.g. AHP method)

Evaluation of the portfolio P according to criterion *j* is obtained by multiplying the share of each stock s_i in the portfolio with the evaluation of stock *i* according to criterion *j*:

$$C_j(P) = \sum_{i=1}^{N} x_i C_j(s_i)$$

Where x_i is the share invested in s_i in the portfolio *P*. Naturally, $\sum_{i=1}^{N} x_i = 1$.

For each criterion C_j $(j = 1,2,...,n)$ preference functions are defined as in the PROMETHEE method where indifference (q) and preference (p) thresholds are certain numbers from the interval $\left[0, C_j(\overline{P}) - C_j(\underline{P})\right]$, i.e. the following is applicable

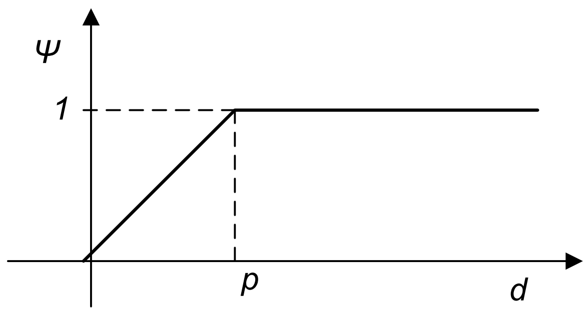
$0 \le q, p \le C_j(\overline{P}) - C_j(\underline{P})$.

By analogy, if Gauss criterion is used the criterion is also applicable to the parameter *s.* Naturally, $q \le p$ is always true. Let's assume, which is in line with the economic significance of such thresholds, that $p \in \left[q, \frac{C_j(\overline{P}) - C_j(\underline{P})}{2} \right]$, i.e. the highest value of preference threshold cannot exceed half the span between the lowest and the highest value according to that criterion.

In this paper we use the linear preference criterion which includes only one threshold.

The linear preference function from the PROMETHEE method has the following form:

$$\Psi(d) = \begin{cases} \frac{d}{p} & 0 \le d < p \\ 1 & d \le p \end{cases}$$



Picture 1. Linear function from PROMETHEE

Where *d* is the difference in evaluation of the two alternatives according to a certain criterion. In our case when portfolio *P* is compared to antiideal $(\underline{P})$, i.e. when we calculate $\Phi_j^+(P)$ difference *d* stands for the "distance" from antiideal (according to criterion *j*), i.e. the higher the difference the preference is closer to 1. Therefore,

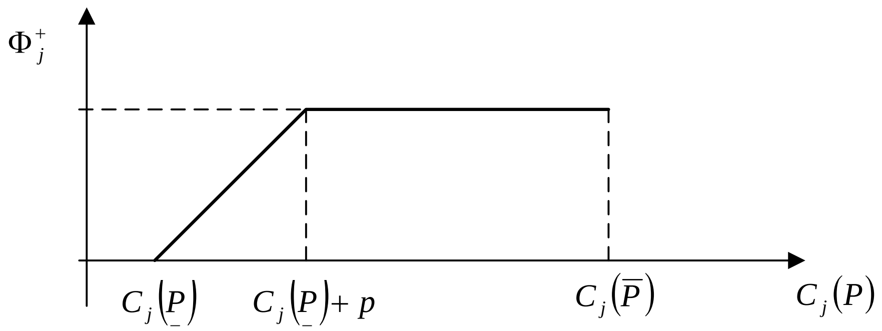
$d_j(P) = C_j(P) - C_j(\underline{P}) = \sum_{i=1}^{N} x_i C_j(s_i) - C_j(\underline{P})$. Since for every P $C_j(P) \geq C_j(\underline{P})$ is always true, it means that $d_j(P) \geq 0$. is also always true. Therefore,

$$\Phi_j^+(P) = \Psi_j(P, \underline{P}) = \begin{cases} \dfrac{C_j(P) - C_j(\underline{P})}{p} & 0 \leq C_j(P) - C_j(\underline{P}) < p \\ 1 & C_j(P) - C_j(\underline{P}) \geq p \end{cases}$$

The value of the positive flow will be higher if difference $d_j(P) = C_j(P) - C_j(\underline{P})$ is higher, i.e., when it exceedes *p* it will be 1. If we wish to present the positive flow as the portfolio P function we have:

$$\Phi_j^+(P) = \begin{cases} \dfrac{C_j(P) - C_j(\underline{P})}{p} & C_j(P) \leq C_j(\underline{P}) + p \\ 1 & C_j(P) > C_j(\underline{P}) + p \end{cases}$$

or graphically:

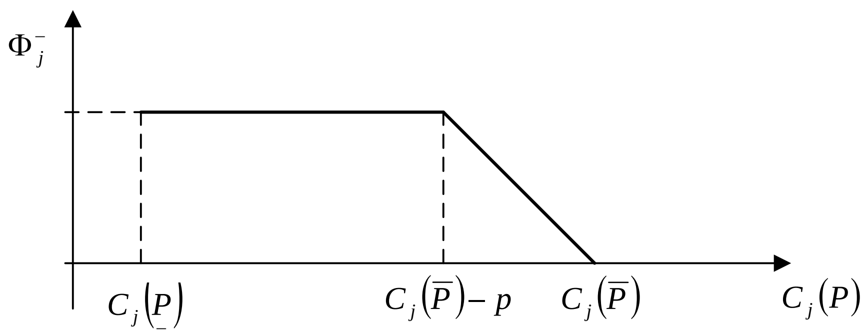


Picture 2. Positive flow

The picture clearly indicates that the positive flow is higher if $C_j(P)$ is higher (with the assumption that all criteria are maximised).
In analogy, the negative flow $\Phi_j^-(P)$ is considered for the difference of the portfolio *P* from the ideal $(\overline{P})$. The smaller the distance $d_j(P)$ the better the portfolio. Since $C_j(\overline{P}) \geq C_j(P)$ is true for all possible portfolios *P* the difference $d_j(P)$ is in that case defined as: $d_j(P) = C_j(\overline{P}) - C_j(P)$, so that the negative flow has the form:

$$\Phi_j^-(P) = \begin{cases} \dfrac{C_j(P) - C_j(\underline{P})}{p} & C_j(P) > C_j(\overline{P}) - p \\ 1 & C_j(P) \leq C_j(\overline{P}) - p \end{cases}.$$

graphically:



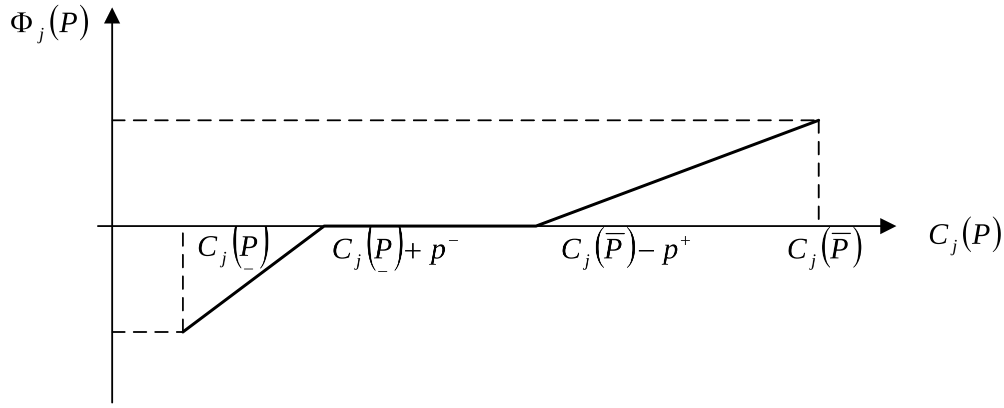
Picture 3. Negative flow

Therefore, the better the portfolio *P* according to criterion *j,* i.e. the higher $C_j(P)$ (where the criterion has to be maximised) the smaller $\Phi_j^-(P)$.

Finally, the net flow of the portfolio *P* is calculated as the difference, i.e. $\Phi_j(P)=\Phi_j^+(P)-\Phi_j^-(P)$, so we have:

$$\Phi_j(P)=\begin{cases}\dfrac{C_j(P)-C_j(\underline{P})-p^-}{p^-}, & C_j(P)\le C_j(\underline{P})+p^-\\ 0, \quad C_j(\underline{P})+p^-<C_j(P)\le C_j(\overline{P})-p^+ & \\ \dfrac{p^++C_j(P)-C_j(\overline{P})}{p^+}, & C_j(P)>C_j(\overline{P})-p^+\end{cases}$$

The graphic presentation of the net flow according to criterion *j* is given in picture 4, where different threshold values for ”distance” from the ideal and antiideal are chosen.



Picture 4. Net flow

Finally, the optimal portfolio is one that solves:

$$Max\,\Phi(P)$$

subject to;

$$\sum_{i=1}^{N} x_i = 1 \text{ i } 0 \le x_i \le x_{M\,i},$$

where

$$\Phi(P)=\sum_{j=1}^{N} w_j\Phi_j(P),$$

$$P = X_P^{'}S, \qquad X_P=(x_1,x_2,\ldots,x_N), \quad S_P=(s_1,s_2,\ldots,s_N),$$

x_i : proportion invested in share $i\ (i=1,2,\ldots,N)$ in portfolio *P,*

$x_{M\,i}$ maximum proportion to invest in share *i* in portfolio *P*,

N is the number of pre-selected shares which can be included in portfolio *P*.

4. APPLICATION TO CROATIAN CAPITAL MARKET

Using the above model we subsequently calculate the optimal portfolios on the Croatian capital market. From the total number of securities quoted on the Zagreb stock exchange in 2007 a sample of ten shares has been separated. The shares sample contains the ten shares with the highest weights from CROBEX index in 2007: INA-R-A, ADRS-P-A, ATPL-R-A, ERNT-R-A, PODR-R-A, IGH-R-A, ZABA-R-A, DLKV-R-A, CROS-R-A, THNK-R-A.

First, absolute values of criteria, like return, risk, price to earning value ratio (P/E), price to book value ratio (P/B) and price to sale value ratio (P/S) are included in our multicriterial model for selecting optimal portfolio. The risk measure we used is the lower semi-variance (testing the yield distribution it turned out that the yields do not have normal distribution and therefore the variance is not an adequate risk measure).

Table 1. Stock's absolute values for the constructed criteria & Table 2. Stock's relative values for the constructed criteria

	E(R)	Lover semi-variance	P/E	P/BV	P/S
	max	min	min	min	min
INA-R-A	-0,8635	3,0092	24,0000	2,2000	1,2000
ADRS-P-A	-0,2910	2,0545	14,0000	1,6000	3,1000
ATPL-R-A	3,2459	5,6079	23,3000	5,1000	3,9000
ERNT-R-A	-0,5970	2,4582	19,8000	3,4000	2,7000
PODR-R-A	-0,7195	1,8207	55,2000	1,8000	0,8000
IGH-R-A	3,6542	8,6027	38,8000	4,8000	3,0000
ZABA-R-A	-0,0817	3,1835	31,4000	3,4000	7,7000
DLKV-R-A	1,8973	4,3163	97,6000	12,2000	2,8000
CROS-R-A	0,5450	3,2167	39,6000	3,5000	2,1000
THNK-R-A	1,3197	6,4170	40,9000	4,6000	1,8000

	E(R)	Lover semi-variance	P/E	P/BV	P/S
	max	min	min	min	min
INA-R-A	-0,8635	3,0092	0,2051	0,7857	0,9131
ADRS-P-A	-0,2910	2,0545	0,1196	0,5714	2,3588
ATPL-R-A	3,2459	5,6079	0,2085	0,5167	0,4918
ERNT-R-A	-0,5970	2,4582	1,1092	1,1724	0,8438
PODR-R-A	-0,7195	1,8207	1,4568	0,7407	0,4167
IGH-R-A	3,6542	8,6027	0,4631	0,6877	1,5957
ZABA-R-A	-0,0817	3,1835	1,0680	1,1240	1,3750
DLKV-R-A	1,8973	4,3163	1,1650	1,7479	1,4894
CROS-R-A	0,5450	3,2167	0,7952	0,9859	1,0244
THNK-R-A	1,3197	6,4170	0,4882	0,6590	0,9574

In order to take into consideration the behaviour of investors, we proceed to the change of weights. Table 3 shows weights of seven possible scenarios. We note, that this application is a simple illustration of the proposed approach. No real decision-maker is implied.

Table 3. Weight of each criterion

Criterion	Mean return	Lover semi-variance	P/E	P/B	P/S
Scenario 1	0.10	0.10	0.30	0.30	0.20
Scenario 2	0.10	0.10	0.20	0.30	0.30
Scenario 3	0.10	0.10	0.30	0.20	0.30
Scenario 4	0.10	0.10	0.40	0.20	0.20
Scenario 5	0.10	0.10	0.20	0.40	0.20
Scenario 6	0.10	0.10	0.20	0.20	0.40
Scenario 7	0.30	0.30	0.20	0.10	0.10

Next we select optimal portfolio using relative value of P/E, P/B,P/S criteria. For criteria exclusively derived from data obtained from capital market like expected return, risk, stock liquidity there is no sense to use relative criteria values (for example, investor always prefer stock with higher stock liquidity no matter what the mean value of stock liquidity of the sector is).

By programming in MATLAB we get solutions given in table 4 and 5. Firstly, an optimal portfolio is calculated without maximum proportion constraint, and then taking into consideration maximum proportion constraint $x_M = 0.7,\ x_M = 0.5,\ x_M = 0.3,\ x_M = 0.2.$

Table 4. Optimal portfolios with relative criteria values

	INA-R-A	ADRS-P-A	ATPL-R-A	ERNT-R-A	PODR-R-A	IGH-R-A	ZABA-R-A	DLKV-R-A	CROS-R-A	THNK-R-A	Return in the next week
	Scenario 1										
x_M=1	0,0000	0,0000	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,8186
x_M=0,7	0,0000	0,0000	0,7000	0,0000	0,0000	0,3000	0,0000	0,0000	0,0000	0,0000	3,2379
x_M=0,5	0,0000	0,0000	0,5000	0,0000	0,0000	0,4666	0,0000	0,0000	0,0000	0,0334	4,7460
x_M=0,3	0,3000	0,3000	0,3000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,1000	-0,9274
x_M=0,2	0,2000	0,2000	0,2000	0,0000	0,0000	0,2000	0,0000	0,0000	0,0000	0,2000	1,9242
	Scenario 2										
x_M=1	0,0000	0,0000	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,8186
x_M=0,7	0,0000	0,0000	0,7000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,3000	2,2963
x_M=0,5	0,0000	0,0000	0,5000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,5000	3,2814
x_M=0,3	0,3000	0,0000	0,3000	0,0000	0,1000	0,0000	0,0000	0,0000	0,0000	0,3000	0,8867
x_M=0,2	0,2000	0,2000	0,2000	0,0000	0,2000	0,0000	0,0000	0,0000	0,0000	0,2000	-0,2323
	Scenario 3										
x_M=1	0,0000	0,0000	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,8186
x_M=0,7	0,3000	0,0000	0,7000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-0,3192
x_M=0,5	0,5000	0,0000	0,5000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-1,0778
x_M=0,3	0,3000	0,1000	0,3000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,3000	0,7916
x_M=0,2	0,2000	0,2000	0,2000	0,0000	0,0000	0,0000	0,0000	0,0000	0,2000	0,2000	-0,7169
	Scenario 4										
x_M=1	0,0000	0,0000	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,8186
x_M=0,7	0,3000	0,0000	0,7000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-0,3192
x_M=0,5	0,5000	0,0000	0,5000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-1,0778
x_M=0,3	0,3000	0,3000	0,3000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,1000	-0,9274
x_M=0,2	0,2000	0,2000	0,2000	0,0000	0,0000	0,2000	0,0000	0,0000	0,0000	0,2000	1,9242
	Scenario 5										
x_M=1	0,0000	0,0000	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,8186
x_M=0,7	0,0000	0,0000	0,7000	0,0000	0,0000	0,3000	0,0000	0,0000	0,0000	0,0000	3,2379
x_M=0,5	0,0000	0,0000	0,5000	0,0000	0,0000	0,4666	0,0000	0,0000	0,0000	0,0334	4,7460
x_M=0,3	0,3000	0,3000	0,3000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,1000	-0,9274
x_M=0,2	0,2000	0,2000	0,2000	0,0000	0,2000	0,0000	0,0000	0,0000	0,0000	0,2000	-0,2323
	Scenario 6										
x_M=1	0,0000	0,0000	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,8186
x_M=0,7	0,0000	0,0000	0,7000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,3000	2,2963
x_M=0,5	0,5000	0,0000	0,5000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-1,0778
x_M=0,3	0,3000	0,0000	0,3000	0,0000	0,1000	0,0000	0,0000	0,0000	0,0000	0,3000	0,8867
x_M=0,2	0,2000	0,0000	0,2000	0,0000	0,2000	0,0000	0,0000	0,0000	0,2000	0,2000	-0,5267
	Scenario 7										
x_M=1	0,0000	0,0000	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,8186
x_M=0,7	0,0000	0,0000	0,7000	0,0000	0,0000	0,3000	0,0000	0,0000	0,0000	0,0000	3,2379
x_M=0,5	0,0000	0,0000	0,5000	0,0000	0,0000	0,4711	0,0000	0,0047	0,0000	0,0242	4,7148
x_M=0,3	0,0000	0,0000	0,3000	0,0000	0,0000	0,3000	0,0000	0,1000	0,0000	0,3000	4,2393
x_M=0,2	0,2000	0,2000	0,2000	0,0000	0,1272	0,0000	0,0000	0,0000	0,2000	0,0728	-1,6892

Table 5. Optimal portfolios with absolute criteria values

	INA-R-A	**ADRS-P-A**	**ATPL-R-A**	**ERNT-R-A**	**PODR-R-A**	**IGH-R-A**	**ZABA-R-A**	**DLKV-R-A**	**CROS-R-A**	**THNK-R-A**	**Return in the next week**
	Scenario 1										
$x_M=1$	0,0000	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-2,8507
$x_M=0,7$	0,3000	0,7000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-2,8877
$x_M=0,5$	0,5000	0,5000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-2,9124
$x_M=0,3$	0,3000	0,3000	0,2907	0,1093	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-1,6596
$x_M=0,2$	0,2000	0,2000	0,2000	0,2000	0,0000	0,0399	0,0000	0,0000	0,1601	0,0000	-1,6138
	Scenario 2										
$x_M=1$	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-2,9741
$x_M=0,7$	0,7000	0,3000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-2,9370
$x_M=0,5$	0,5000	0,5000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-2,9124
$x_M=0,3$	0,3000	0,3000	0,1000	0,0000	0,3000	0,0000	0,0000	0,0000	0,0000	0,0000	-2,2355
$x_M=0,2$	0,2000	0,2000	0,1843	0,0000	0,2000	0,0614	0,0000	0,0000	0,1543	0,0000	-1,5159
	Scenario 3										
$x_M=1$	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-2,9741
$x_M=0,7$	0,7000	0,3000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-2,9370
$x_M=0,5$	0,5000	0,5000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-2,9124
$x_M=0,3$	0,3000	0,3000	0,2911	0,0801	0,0287	0,0000	0,0000	0,0000	0,0000	0,0000	-1,6738
$x_M=0,2$	0,2000	0,2000	0,2000	0,1470	0,1071	0,0640	0,0000	0,0000	0,0820	0,0000	-1,1927
	Scenario 4										
$x_M=1$	0,0000	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-2,8507
$x_M=0,7$	0,0580	0,7000	0,2420	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-1,9698
$x_M=0,5$	0,2301	0,5000	0,2699	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-1,8889
$x_M=0,3$	0,3000	0,3000	0,2907	0,1093	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-1,6596
$x_M=0,2$	0,2000	0,2000	0,2000	0,2000	0,0000	0,0398	0,0000	0,0000	0,1602	0,0000	-1,6142
	Scenario 5										
$x_M=1$	0,0000	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-2,8507
$x_M=0,7$	0,3000	0,7000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-2,8877
$x_M=0,5$	0,5000	0,5000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-2,9124
$x_M=0,3$	0,3000	0,3000	0,1000	0,0000	0,3000	0,0000	0,0000	0,0000	0,0000	0,0000	-2,2355
$x_M=0,2$	0,2000	0,2000	0,2000	0,0000	0,2000	0,0478	0,0000	0,0000	0,1522	0,0000	-1,6151
	Scenario 6										
$x_M=1$	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-2,9741
$x_M=0,7$	0,7000	0,0000	0,0000	0,0000	0,3000	0,0000	0,0000	0,0000	0,0000	0,0000	-2,6517
$x_M=0,5$	0,5000	0,0000	0,0000	0,0000	0,5000	0,0000	0,0000	0,0000	0,0000	0,0000	-2,4369
$x_M=0,3$	0,3000	0,3000	0,0000	0,0000	0,3000	0,0933	0,0000	0,0000	0,0067	0,0000	-1,5176
$x_M=0,2$	0,2000	0,2000	0,0000	0,0000	0,2000	0,2000	0,0000	0,0000	0,2000	0,0000	-0,6328
	Scenario 7										
$x_M=1$	0,0000	0,0000	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,8186
$x_M=0,7$	0,0000	0,0000	0,7000	0,0000	0,0000	0,3000	0,0000	0,0000	0,0000	0,0000	3,2379
$x_M=0,5$	0,0000	0,0000	0,5000	0,0000	0,0000	0,4859	0,0000	0,0000	0,0141	0,0000	4,6649
$x_M=0,3$	0,1217	0,3000	0,2783	0,3000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-1,4012
$x_M=0,2$	0,1726	0,2000	0,2000	0,2000	0,0000	0,0274	0,0000	0,0000	0,2000	0,0000	-1,8149

From the tables above we can see that portfolios obtained using absolute and relative criteria values in the multicriterial model are significantly different. Furthermore we took returns of pre-selected stocks in subsequent period and we calculated the return of each optimal portfolio in subsequent period.

The results presented in the tables also indicate that every portfolio obtained using relative criteria values in model have higher return in the subsequent period than portfolios obtained using absolute criteria values.

5. SUMMARY AND CONCLUSIONS

The aim of this paper was to point out one of the problems that arise when the multicriteria model is applied to selection of optimal portfolios. The problem researched in this paper refers to the comparability of stock from different sectors according to particular criteria. Namely, stocks of different sectors cannot be compared on those criteria the mean value of which is significantly different in different sectors. For such criteria we introduce relative criteria value defined as ratio of absolute value criteria for certain stock and mean criteria value of corresponding sector. Aiming at proving that stock evaluation based on the relative values of “problematic” criteria gives better results than stock evaluation based on the absolute values of those criteria we select optimal portfolio using absolute and relative criteria values respectively and then we compare optimal portfolios selected by absolute and relative criteria values.

The comparison of the obtained portfolios was carried out by taking the returns of pre-selected stocks in subsequent period and by calculating the return of all optimal portfolios in subsequent period. The results obtained form the Croatian capital market indicate that every portfolio obtained using relative criteria values in model has higher return in subsequent period than the portfolios obtained using absolute criteria values.

From all previously stated facts we can conclude that using relative values of criteria proposed in this paper in multicriterial models gives better results (in sense of return) than using absolute criteria values.

REFFERENCES

Aljinović Z., Marasović B., Tomić-Plazibat N., (2005): **The Selection of the Optimal Portfolio on the Croatian Capital Market,** *Proceedings of Sixth International Conference on Enterprise in Transition (extended abstract, 299-301, full text on CD ROM)*

Anderson, P. L., (2004): **Business Economics and Finance with MATLAB, GIS, and Simulation Models**, MATLAB & Simulink Based Books, Chapman&Hall/CRC

Benninga, S., (2000): **Financial Modeling**, The MIT Press, Cambridge

Black, F., (1972): **Capital Market Equilibrium with Restricted Borrowing**, *Journal of Business*, 45: 444 – 455

Bouri A., Martel J. M., Chabchoub H, (2002).: **A Multi-criterion Approach For Selecting Attractive Portfolio,** *Journal Of Multi-Criteria Decision Analysis* 11:269-277

Brans JP, Mareschal B, Vinike P. (1984): **PROMETHEE:A new family of outranking methods in multicriteria analysis.** In Operational Research'84, Brans JP (ed.), North-Holland: Amsterdam; 477–490

Cloquette, J.F., Gerard, M., Hadhri, M., (1995): **An empirical analysis of Belgian daily returns using GARCH models**, Cahiers Economiques de Bruxelles 418, 513-535.

Khoury N, Marte JM, Veilleux M. (1993):. **Methode multicritere de selection de portefeuilles indiciels interantionaux**. Acualite Economique 69(1): 171–190.

Konno, H., Waki, H., Yuuki A., (2002): ***Portfolio Optimization under Lower Partial Risk Measures***, Institute of Economic Research, Kyoto University

Markowitz H. M., (1959): **Portfolio Selection: Efficient Diversification of Investment**, Yale University Press: New Haven, CT.

Prohaska, Z., (1996): **Analiza vrijednosnih papira**, Infoinvest, Zagreb

Rockafaller, R. T. I Uryasev, S., (2000): **Optimization of Conditional Value-at-Risk,** *Journal of Risk*, 2, 21-41

Venkataraman P., (2002): **Applied Optimization with MATLAB Programming**, John Wiley & Sons, New York

The Zagreb Stock Exchange (2007): **History Trading 2007**, http://www.zse.hr/history.php?sessionId=62b230da7398c66c9575bdb8625d416e