Long-Short Portfolio

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Abstract

Long-short strategies are one of the most successful tools, applied by hedge funds manager. One under-evaluated stock is bought (long position) and an over-evaluated stock is sold (short position) at the same time. After a short term, when the values of the stocks are as expected, profit can be realized by a closing transaction. The possibility to find first obvious over- and under-evaluated stocks depends on the number of participants in this markets. While the hedge funds strategies become more popular, the chance to achieve profit by this strategies is shrinking.

Therefore two models to generate long-short portfolios are proposed. By this approaches a portfolio A for the long- and a portfolio B for the short position were computed. The difference of the values of A and B is designed to oscillate from negative to positive and reverse. This behavior of oscillating or mean reverting stock prices was stated by e.g. E. Fama and K. R. French (1988). Mean reversion of portfolios can offer the possibility of statistical arbitrage. The proposed linear models were tested by stocks of the Tokyo stock exchange. The results seem to be applicable and show an additional advantage of low systematic risk.

Key-Words: Portfolio management, hedge funds, long-short-strategy, mixed integer linear optimization, statistical arbitrage, single-index-model, beta-factor, mean reversion

1. Introduction

Besides active and passive portfolio management, in recent years portfolios were constructed, which produce profit in bearish markets, too. Examples are the so called Hedge-Funds². This funds claim to be hedged, because of e.g. the long-short-strategies they use to be market independent. By this strategy, investors can earn in bullish and in bearish markets.³ To apply this strategy means e.g. to buy an asset with underestimated value and to sell an asset with overestimated value at the same time. If the expectations concerning the mean of the both assets are correct, the price of the both assets will move back to the mean in some weeks or months. Then, the closing transaction offers an market independent return. The long-short-strategy can also be applied, if two assets A and B seem to have equal mean value. In this case, the market beliefs, that the value of the both is comparable. The difference between the price of A and B is temporary and offers another application of a long-short-strategy.

Compared with traditional portfolio management⁴, long-short-strategies do not interpret the expected return μ itself but e.g. the temporary deviation of this μ as valuable chance. Therefore a long-short-position will be closed after a short time interval when the deviation is disappeared. There is no long-term investment planned.

The knowledge of the mean reverting behavior of stock prices offers a kind of statistical arbitrage. Like in the case of the arbitrage, to achieve profit, it is important to be the first, who recognizes the deviation of the equilibrium or of the mean. Due to the increasing number of

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³ If the market is either bullish nor bearish, this strategy seem to be less successful.
⁴ see Markowitz H. (1952); Sharp W. F. (1964)
participants in the markets, searching for arbitrage, the possibility to apply successful a long-
short-strategy becomes smaller.

Beside taxation often reduces the profit of investors. To estimate the investors portfolio
selection behavior, the system of taxation must be respected. The after tax profit of capital
gains is dependent on, whether capital gains are taxed and losses are deduced without
restriction at the same rate or not or whether capital gains are taxed only if realized within 12
month like in Germany etc.. Taxation can produce a wide range of distortion in portfolio
behavior. The proposed approaches below occur in absence of non-neutral distortioning
taxation⁵.

In the capital market theory the above described behavior of stock prices is known as
“Mean Reversion⁶”. Whether the mean reverting process stays or not, when portfolios instead
of single stocks are regarded, is unknown. If mean reversion exists, it must be observable and
util for statistical arbitrage when the mean reverting behavior remains in the near future.

To investigate mean reversion of portfolios, two models where designed. Instead of two
stocks like in Figure 1 for the long- and the short position, two portfolios A and B must be
found by the models. The difference of the values of A and B should be oscillating from
negative to positive and reverse. By this, the mean of the difference of the two portfolios can
be expected as about zero. The value of the portfolio (+A-B) should revert to its mean of zero
in a fixed time interval. The two models will be called “Max Tau” and “Max Sum” in the
following. After the introduction of the models, they will be tested by empirical data of the
Japanese stock exchange.

![Strategy: Mean Reversion](Long-short02.dsf 0605)

**Fig. 1: Mean Reversion**

### 2. Long-short portfolio: Max Tau

Portfolios A and B with the behavior shown by the two stocks in Figure 1, can be
designed by a linear mixed integer model. For this purpose the time \( t=1, \ldots, T \) was divided
into subsets \( T_k \) \((k=1, \ldots, m)\) which may not be exhaustive.

The values \( v_{it} \) of the \( i=1, \ldots, n \) assets at time \( t=1, \ldots, T \) are used directly as parameters.
The budget \( C \), divided by the value \( v_{iT} \) of asset \( i \) at the time \( T \) shows the number of assets \( x_i \)

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⁵ A wide range of tax-induced distortions are discussed in Auerbach, A. J., Hines Jr., J. R. (2002) and Poterba, J.

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(i=1, ..., n) which can be bought at T. The product \( x_i \cdot v_{iT} \) is the amount invested in asset i. The sum of this amount is the value of the long-short portfolio (+A-B) or the distance \( d_t \) (t= 1, ..., n) between the values of A and B (see inequalities (2a)). The budget C in the model is equal for every asset but should be individual fixed for every asset dependent on the possibilities to buy or sell this asset. Portfolio A will contain stocks with \( x_i > 0 \) and portfolio B stocks with \( x_i < 0 \).

If the distances \( d_t \) (t\( \in \) \( T_k \)) between the values of the two sub-portfolios should be positive, the distances \( d_t \) (t\( \in \) \( T_{k+1} \)) should be negative and reverse. In this subsets, the distance level \( \tau \) resp. \( -\tau \) should be at least once met. This distance level \( \tau \) will be maximized by the objective function (see (1)).

In equality (2b) the variables \( c_i^- \) and \( c_i^+ \) (i=1, ..., n) measure as slack-variables what amount of asset i should be sold (\( c_i^- \)) or bought (\( c_i^+ \)). The sum of both is in inequality (2c) restricted at least to the twice of the budget C. This means, that most of the long positions are financed by short-selling or that the sum of the investments is closed to zero in T.

To get high deviations between the values of the two sub-portfolios, the objective function of the model must

\[
\text{maximize } \tau \quad (1)
\]

under the conditions

\[
\sum_{i=1}^{n} x_i \cdot v_{iT} = d_t \quad \text{with} \quad -C/v_{iT} \leq x_i \leq C/v_{iT}, \quad (i = 1, ..., n) \quad (2a)
\]

\[
v_{iT} x_i + c_i^- - c_i^+ = 0, \quad (i = 1, ..., n) \quad (2b)
\]

\[
\sum_{i=1}^{n} c_i^- + c_i^+ \leq 2C \quad (2c)
\]

and

\[
d_t \geq (\delta_t^o - 1) M + \tau, \quad (t \in T_k, \quad k = 1, 3, 5, ..., m-1) \quad (3a)
\]

\[
d_t \leq \tau - \varepsilon + \delta_t^o M, \quad (t \in T_k, \quad k = 1, 3, 5, ..., m-1) \quad (3b)
\]

\[
\sum_{t \in T_k} \delta_t^o \geq 1, \quad (k= 1, 3, 5, ..., m-1) \quad (3c)
\]

\[
d_t \leq (1- \delta_t^u) M + \tau, \quad (t \in T_k, \quad k = 2, 4, 6, ..., m) \quad (4a)
\]

\[
d_t \geq \tau + \varepsilon - \delta_t^u M, \quad (t \in T_k, \quad k = 2, 4, 6, ..., m) \quad (4b)
\]

\[
\sum_{t \in T_k} \delta_t^u \geq 1, \quad (k= 2, 4, 6, ..., m) \quad (4c)
\]

with \( \varepsilon \): small number, M: big number.

The restrictions (3a) - (3c) force the model, to produce at least once a distance \( d_t \geq \tau \) (t\( \in \) \( T_k \)) in subset \( T_k \). The restrictions (4a) - (4c) guaranties, that the solution has in the following subset \( T_{k+1} \) also at least once a distance \( d_t \leq -\tau \) (t\( \in \) \( T_{k+1} \)). If the distance \( \tau \) resp. \( -\tau \) is met, the binary variables \( \delta_t^o \) resp. \( \delta_t^u \) in (3) resp. (4) become equal one. The inequality (3c) resp. (4c) counts this cases. Within two subset \( T_k \cup T_{k+1} \) the distance \( d_t \) of the two portfolios A and B must be about zero at least once (see Figure 2 – left side).
To reduce the management costs of the portfolio, additional restrictions to the number of stocks can be integrated:

\[
\begin{align*}
    x_i - M \delta_i^+ &\leq 0 \quad (i=1, \ldots, n) \quad \text{and} \quad \sum_{i=1}^{n} \delta_i^+ \leq z^+ \tag{5a} \\
    x_i + M \delta_i^- &\geq 0 \quad (i=1, \ldots, n) \quad \text{and} \quad \sum_{i=1}^{n} \delta_i^- \leq z^- \tag{5b}
\end{align*}
\]

with

\[
\begin{align*}
    \delta_i^+, \delta_i^- &\colon \text{binary variables} \\
    z^+, z^- &\colon \text{max. number of assets long (}z^+\text{) or short (}z^-\text{)} \\
    M &\colon \text{big number.}
\end{align*}
\]

For an empirical test of the “Max Tau” long-short model, data from the Japanese capital market were used. The data base were the 86 biggest Japanese stocks which were listed in the stock exchange in Tokyo throughout the period from September 5th 1988 until November 1st 1999 (some selected stocks are listed in Table 1). For the optimization, only the daily stock values of this stocks from January 3rd 1994 until December 31th 1998 were used. The total number of values \(v_{it}\) for each of the 86 stocks were 1304. In the following 10 months (until the November 1st 1999) after the optimization or the following 217 values \(v_{it}\) were used to control the behavior of the optimized portfolio. If the models produce portfolios A and B which revert to their mean value, this behavior must be shown in this 10 month, too.

For the optimization the time was divided into 5 subsets \(T_k\) each with about 261 values \(v_{it}\). One subset corresponds with one year. To determine the positive and negative bounds of the variables \(x_1, \ldots, x_n\), an uniform max. budget \(C = 0.5 \text{ Mio Yen}\) was used for every asset in (2a). The absolute Budget of long- and short positions is \(2C \leq 1 \text{ Mio Yen}\). Figure 3a illustrates the solution of an example in which the number of assets in the portfolios A resp. B was not constrained by inequalities (5). The realization of such portfolios can fails, due to the rare shortselling possibilities in financial markets. Therefore in Figure 3b the number of assets which should be sold was restricted to \(z^- = 1\) (see (5b)). The difference \(d_t\) between the Portfolio A and B was in the example of Figure 3a at least \(\tau = 0.161 \text{ Mio Yen}\) in each subset \(T_k\) and in the example of Figure 3b at least \(\tau = 0.123 \text{ Mio Yen}\). The vertical lines divide the 5 subsets \(T_k\). Each subset contains about 260 days resp. values \(v_{it}\). The smaller subset at the right side is related to the 10 month after the 31.12.1998. The optimization process was in both cases aborted after some hours of consumed CPU-time.

The unrestricted portfolio contains 10 assets long and 9 assets short and the restricted portfolio 3 resp. 1 asset. The blue chart is the value of the portfolio A (long position) and the
red chart of the portfolio B (short position). At T = 1.1.1999 the value of both portfolios is 0.5 Mio (see horizontal dotted line). After this date, the distance of the value of the two portfolios is growing in Figure 3a. In Figure 3b this distance shows in the test-interval a similar behavior like in the optimization periods. The chart of the long-short portfolio crosses the zero-line and offers a possibility for statistical arbitrage.
Within the optimized time interval the solutions of the model “Max Tau” offer portfolios with the expected behavior (see Figure 2). The CPU-time for finding an acceptable good solution (without knowing whether it is the optimal solution) was high, due to the amount of binary variables and structures used. Therefore, a second algorithm “Max Sum” without binary variables was tested, although the objective function is not searching high extreme values of the distance $d_i$ in every subset $T_k$.

3. Long-short portfolio: Max Sum

The following approach to find a long-short portfolio is linear. Like in the model “Max Tau”, subsets of the time are used. For each subset $T_k$ ($k=1, ..., m$) the sum of distances $d_t$ ($t \in T_k$) is computed. This sum should be positive in the first subset, negative in the second subset and positive in the following and so on (see Figure 2 - right side). Like in the model “Max Tau”, the sum of distances $d_t$ should be positive with a value of at least $+\text{Sum}$ in the first subset and negative and a value of at most $-\text{Sum}$ in the second subset and so on.

To achieve portfolios A and B the objective function

$$\text{maximize} \ \text{Sum}$$

under the conditions (2a)-2(c) and

$$\sum_{t \in T_k} d_t \geq + \text{Sum}, \quad (k = 1, 3, 5, ..., m-1)$$

(7a)

$$\sum_{t \in T_k} d_t \leq - \text{Sum}, \quad (k = 2, 4, 6, ..., m).$$

(7b)

To reduce the management costs of the portfolio, restrictions like above can be integrated (see (5)).

For an empirical test of the model “Max Sum” data from the Japanese capital market were used like in the empirical test of the model “Max Tau”.

In Figure 4a the solution without restriction is shown and in Figure 4b the number of short positions was restricted to one. The shape of the chart of the long-short portfolio in Figure 4a is similar to the shape of the chart in Figure 3a. This is surprising, because of the different stocks the two long-short portfolios contain (see Table 1). In the case of Figure 3b and 4b, the portfolio B is identical. It contains the stock MITSUBISHI ELECTRIC CORP. Therefore, the shapes of the long-short portfolios are similar. The CPU-time for solving the two examples was below one second.

Like above, the restricted long-short portfolio (Figure 4b) gives a better forecast of the behavior of the long-short portfolio compared with the unrestricted case. The value of the objective function is an aggregated value ($\text{Sum} = 37.5 \text{ Mio} \text{ resp.} 22.4 \text{ Mio}$). To compare this value with $\tau$, it is better to use the average value, although the average value will be smaller than the maximum value $\tau$. If each subset $T_k$ contains 260 time intervals, the average distance $d_t$ would be at least 0.144 Mio resp. 0.086 Mio Yen. Compared with the two $\tau$ values of the examples above, the solution of the “Max Sum” model seem to produce comparable results.

To see the influence of the size of the subsets $T_k$, only 130 time intervals where used in the example of Figure 4c. Now, the chart of the long-short portfolio has a rapid changing
direction. In every \( T_k \) several positive and negative extremes can be observed. The deviations are within the interval \( +/- 0.1 \) Mio (see red horizontal lines). The subsets \( T_k \) seem to be too small selected.

**LS- Portfolio: max SUM**

Budget: 1 Mio; long: 2; short: 3; sum= 37.5  Mio; scaling: 1/1000

**Fig. 4a: Model “Max Sum” without restrictions**

**LS- Portfolio: max SUM**

Budget: 1 Mio; long: 3; short: 1; sum= 22.4  Mio; scaling: 1/1000

**Figure 4b: Model “Max Sum” with one asset sold**
4. Mean-variance- and long-short portfolios

The above designed long-short portfolios do not have a long term strategy and therefore the mean-variance space is not the right frame for this instruments. Nevertheless it seems to be interesting, to regard the traditional measurements for risk like the variance or the $\beta$.

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Table 1: Stocks, selected by the portfolios

The mean-variance space of Figure 5 contains some efficient portfolios, an index portfolio (with equal weighted stocks), the 86 stocks, a long-short portfolio with maximized $\alpha$ (while $\beta$=0) and the above computed long-short portfolios (see data in Table 2). The return and the standard deviation refer to returns per day.

Base for long-short portfolio with maximized $\alpha$ (while $\beta$=0) is the single index model of W. F. Sharpe\(^7\). Such portfolios must not be designed heuristically,\(^8\) they can be constructed by linear optimization. This portfolio will produce market independent high returns if the $\alpha$ of the portfolio is high. Therefore the assets with high positive $\alpha_i$ must be bought and the assets with negative $\alpha_i$ must be sold to get a portfolio with high market independent returns.

The unrestricted long-short portfolios discussed above have low variance and low market risk with $\beta \leq 0.1$ (see Table 2). With the condition that only one stock should be sold, the systematic risk is about $\beta = +/- 0.3$. For this cases, the standard deviation of the returns is higher too (see Figure 5 and Table 2). The highest return and risk measured by the standard deviation has the portfolio with maximized $\alpha$. The risk measure $\beta$ for the systematic risk is minimal ($\beta$=0). It is surprising, that portfolios without restrictions on the number of stocks and low market risk ($\beta < 0.1$) have their position closed to the index with a systematic risk of $\beta = 1$.

\(^7\) see Sharpe W. F. (1964).
\(^8\) see Farrell, J. L. (1997), pp. 259ff.
The return of the long-short portfolios of the model “Max Tau” and “Max Sum” is closed to zero, due to their construction which forces the portfolios to produce positive and negative returns. For statistical arbitrage, the times of negative returns can also be used to produce profit. Therefore, the absolute value of the returns would be a more adequate measure of return of this long-short portfolios.

Fig. 4c: Model “Max Sum” with one asset sold and subset with 130 days

Fig. 5: Mean-Variance- and Long-Short-Portfolios
5. Conclusions

Although the test of the long-short portfolios is based on few examples some results and questions were obvious. Statistical arbitrage is not without risk like arbitrage. The desired behavior of the value of the long-short portfolio can be well produced for the optimization time interval. Important to achieve profit is, that the observed or expected relationship or behavior of the two portfolios will also be true for the time of investment. For this interval, the risk to change the behavior can not be reduced to zero. The proposed models do not always produce portfolios with the same mean reverting behavior in the post optimization time. The results show, that mean reversion behavior sometimes seem to be valid for the prices of portfolios, too. Nevertheless some economic reasons based on the economic sectors of the stocks and sub-portfolios etc. would be important for creating trust in this long-short strategies. The models can help to discover such relationships. Compared with traditional portfolio risk measure, some of the long-short portfolios show low standard deviation as well as low β resp. systematic risk.

The branch & bound algorithm CPLEX used for solving the “Max Tau” model was always aborted. The solutions found at this moment were nevertheless interesting. The results of the two models are very similar. If the CPU-time for solving the “Max-Tau” model can not be reduced, the model “Max Sum” seem to be a very good alternative model. This model has the feature to select less stock for the portfolio than the “Max Tau” model or the traditional models which search efficient portfolios.

6. References