Testing the Validity of the J-curve Hypothesis between Brazil and the USA

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Abstract

This paper aims to investigate the validity of the J-curve hypothesis between Brazil and the USA using quarterly data for the period of 1981Q1- 2015Q1. To achieve this aim, the vector error correction (VEC) with cointegration, NARX (non-linear autoregressive exogenous) and ANFIS (adaptive neuro-fuzzy inference system) models are separately applied to strengthen this investigation. NARX and ANFIS, as artificial neural networks (ANN) models, were used for the first time in this study to test the validity of the J-curve hypothesis. It was found that the real exchange rate and income, as the independent variables, and the trade balance between Brazil and the USA, as the dependent variable, are cointegrated in the long-run. The empirical findings of all testing models examined in this study indicate that the J-curve hypothesis is not valid between Brazil and the USA. The real depreciations of the Brazilian currency do not make a positive contribution to the trade balance for Brazil.

Resumen

Este trabajo tiene como objetivo investigar la validez de la hipótesis de la curva J entre Brasil y EE.UU Utilizando datos trimestrales para el período 1981Q1-2015Q1. Para lograr este objetivo, los modelos vectoriales de corrección de errores (VEC) con cointegración, NARX (exógeno autoregresivo no lineal) y ANFIS (sistema de inferencia neuro-difusa adaptativa) se aplican por separado para fortalecer esta investigación. NARX y ANFIS, como modelos de redes neuronales artificiales (ANN), se utilizaron por primera vez en este estudio para probar la validez de la hipótesis de la curva J. Se encontró que el tipo de cambio real y el ingreso, como variables independientes, y la balanza comercial entre Brasil y EE.UU, como la variable dependiente, están cointegrados en el largo plazo. Los resultados empíricos de todos los modelos de prueba examinados en este estudio indican que la hipótesis de la curva J no es válida entre Brasil y EE. UU. Las depreciaciones reales de la moneda brasileña no contribuyen positivamente a la balanza comercial de Brasil.

Keywords: J-curve hypothesis, VECM with cointegration, NARX, ANFIS, Brazil, the USA
JEL code: F10, F14, F31

1.- Introduction

Most countries suffer from large and persistent trade deficits that can damage their economies. To counteract these deficits, a country may attempt to use real depreciation on their currency in order to eliminate or mitigate trade deficits within their trade balances. After utilizing real depreciation of its currency, a country's exports become cheaper for consumers abroad. At the same time, that country's domestic consumers also face higher costs for imports. Thus, a country choosing to depreciate its currency is expecting to increase its volumes of exports and decrease the volumes of imports in the long-run, even if its trade-balance worsens in the short-run. But these expectations depend on the responsiveness of exports and imports to the changes in real depreciation. In other words, when the elasticities of demand for imports (e_m) and exports (e_x) highly respond to the levels of real depreciation by meeting the Marshall–Lerner (ML) condition developed by Marshall (1923) and Lerner (1944), the pattern of the initial worsening in a country's trade balance is expected to be followed by an eventual long-term improvement, resembling the letter "J." For this reason, this pattern is known as the "J-curve."

After the introduction of the J-curve concept by Magee (1973), many researchers have been trying to test the validity of the J-curve hypothesis for different countries. However, the empirical findings on the J-curve hypothesis are mixed and vary depending on the countries, the data samples and the methodologies considered in a particular study. Bahmani-Oskoee and Ratha (2004) and Bahmani-Oskooee and Hegerty (2010) comprehensively reviewed the empirical literature on the J-curve hypothesis. The results of these surveys concluded that current empirical evidence does not support a clear answer on the validity of the J-curve hypothesis. While Krugman and Baldwin (1987), Demirden and Pastine (1995), Gupta-Kapoor and Ramakrishnan (1999), Kale (2001), Lal and Lowinger (2002), Gomes and Paz (2005), Halicioglu (2008) found evidences of the J-curve, Rose and Yellen (1989), Shirvani and Wilratte (1997), Bahmani-Oskooee and Brooks (1999), Wilson and Tat (2001), Wilson (2001), Bahmani-Oskooee and Goswami (2003), Akbostanci (2004) and Hsing et al. (2010) failed to find any evidence for some countries.

This study investigates the validity of the J-curve hypothesis between Brazil and the USA. The reasoning behind selecting Brazil as the sample and center country of this study is three-fold. First, the effects of different exchange rate regimes and changing values of Brazilian currency (Real) have long had economic impacts on the country. Specifically, the *crawling peg* exchange rate regime adopted by the Central Bank of Brazil (BCB), as a result of an overvalued currency, was blamed for persistent trade deficits in the 1990s (Gomez and Paz 2005; Papageorgiu et al. 1991; Ribeiro et al. 2007). Moreover, in recent years, the country has also experienced devaluations and the adoption of different exchange rate regimes, such as the *managed floating*, *floating* and *quasi-fixed* exchange rate regimes (Giavazzi 2003; Holland 2006; Pinheiro et al. 2001; Williamson, 2010). Second, the trade balance between Brazil and the USA, as the second largest trading partner of Brazil, has been worsening for Brazil especially in last decade. Third, the appreciations and depreciations of the Real over time make

Brazil a kind of unique lab that gives researchers a chance to observe the results of these fluctuations on the trade balances of the country. All these reasons together raise the importance of testing the validity of the J-curve hypothesis for Brazil with the USA. In other means, the results of this empirical study might be very useful in practice not for only Brazil's policy makers but also for the other developing countries' policy makers intending devaluations to consolidate their trade balances. Because, Brazil has long experience with regards to adoptions of different exchange rate regimes and devaluations so far.

In terms of Brazil, there have been few studies trying to test the validity of J-curve hypothesis but the empirical results that have been gathered are ambiguous. Bahmani-Oskooee and Malixi (1992) tested the hypothesis for some developing countries, including Brazil, and found evidence to support the J-curve hypothesis for Brazil. Gomez and Paz (2005) examined only Brazil, as sample country, and found evidence for the J-curve hypothesis. But, Moura and Da Silva (2005) found no evidence of the J-curve for Brazil which was the only sample country of their study. Hsing (2008) also tested J-curve hypothesis for some Latin American Countries and found no evidence for Brazil. To complicate matters further, Bahmani-Oskooee et al. (2014) tested the hypothesis only between Brazil and the USA for each of the 92 industries between two countries. They aimed to avoid aggregation bias which may occur from using aggregation trade data between Brazil and the rest of the world. They found evidences for 31 industries. Similarly, Mustafa et al. (2015) found evidence of the J-curve for Brazil only in the short-run.

The rest of the paper is organized as follows: Section 2 explains the methodology, section 3 and 4 explain the data set and the empirical models, section 5 discusses the empirical results of VEC with cointegration, NARX and ANFIS models, and section 6 provides the concluding remarks along with recommendations for application and additional research.

2.- Methodology

There is no general consensus between researchers on the best methodology for testing the validity of the J-curve hypothesis. So far, several methodologies have been applied to test the J-curve hypothesis for different sample countries. For example, while Rose and Yellen (1989) used the error correction model (ECM) for the USA with her trading partners, Gupta-Kapoor et al. (1999) used ECM and Impulse Response Functions for Japan. Even when analyzing the same country, different methodologies have been employed between studies. For instance, Bahmani-Oskoee and Ardalani (2006) used Autoregressive Distributed Lag (ARDL) approach with cointegration analysis when analyzing the USA, whereas Demirden and Pastine (1995) used the Vector autoregressive (VAR) approach and Baek et al. (2002) used the ARDL cointegration approach. More recently, Bahmani-Oskoee et al. (2015) used the nonlinear version of ARDL cointegration and ECM for Mexico and 13 trading partner countries of this country.

In this study, we applied VECM with cointegration, NARX and ANFIS models separately to strengthen the investigation of the J-curve hypothesis between Brazil and the USA. ECM methodology was used in many studies testing the J-curve hypothesis. Rose and Yellen (1989) used ECM and could not found any evidence of the J-curve for the USA with her 6 trading partners. Gupta-Kapoor et al. (1999) used ECM and found evidence of the J-curve effects on Japanese trade balance. Alvarez Ude and Gomez (2006) used ECM found no evidence of the J-curve for Argentina with her tarding partners. Bahmani-Oskooee and Ali M. Kutan (2007) used ECM and found evidence of the J-curve for 3 countries in 11. Similarly, Ratha and Kang (2007) used ECM and found the J-curve effects on South Korean trade balance with her trading partners. While VEC model with cointegration has been used in previous studies, this research uses the NARX and ANFIS models, for the first time, to test the J-curve hypothesis. The NARX and ANFIS models, which are set up under the assumption of nonlinear relationship between the variables, are based on artificial neural networks (ANN[1]).

The interest of using ANN in data analyses and ANN models has been growing in economics studies for several years (Angstenberger 1996; Yao and Tan 2000; Qi 2001; Frank and Schmied 2003; Fioramanti 2008; Emam and Min 2009; Chaudhuri and Ghosh 2016). In many cases, it can be difficult to properly model time series because of the complexity of system dynamics. Therefore, applying nonlinear prediction architectures, such as NARX and ANFIS, could be useful in order to improve prediction performance (Mandic 2001; Karray et al. 2004). The systematic of ANN models, inspired by biological neuron, is to estimate the next sample value of a time series without feeding back it to the model's input regressor (Menezes 2006). Additionally, according to Wang, Chau, Cheng and Qiu (2009); Lohani, Kumar and Singh (2012), ANN models may perform better than other forecasting methods in many cases.

NARX[2] is an artificial neural network model employed to predict a value of a time series using the historical data of the same series and the current and historical data of external input series. The standard NARX network is a two-layer feed forward network, with a sigmoid transfer function in the hidden layer and a linear transfer function in the output layer (Gao and Er 2005). This network also uses tapped delay lines to store the historical values of the $x(t)$ and $y(t)$ sequences. Note that the output of the NARX network, $y(t)$, is the current prediction value to the input of the network (through delays) since $y(t)$ is a function of $y(t-1), y(t-2), \dots, y(t-d)$ (Babuška and Verbruggen 2003).

ANFIS is a hybrid mathematical model that compounds fuzzy mathematic and neural network. The main purpose of this model is to determine the relationship between variables by using linguistic variables. The ANFIS[3] model, developed by Jang (1993), is a mechanism, that has the ability to simultaneously learn, adapt and process information.

[1] For the fundamentals of ANN models in forecasting financial and economic time series, see Kaastra and Boyd (1996)

[2] For more information about NARX model see Diaconescu (2008).

[3] For more information about ANFIS model see Jang (1993).

3.- Data Set

In this study, the data used are quarterly figures covering the period of 1981Q1- 2015Q1. Trade balance (TB) was defined as the rate of Brazil's imports divided by the Brazil's exports. The data used to determine TB were obtained from the IMF's Direction of Trade Statistics. The data for the Industrial Production Index (PI), as a proxy of the GDP, were obtained from the database of the Federal Reserve Bank of St. Louis (FED) (2010=100). The real exchange rate (REX) between the Brazilian Real and the USD was calculated by using the nominal exchange rate (NEX) and consumer price indexes ($CPIs$) for all items for both Brazil and the USA (2010=100). NEX was defined as the number of units of the USD per Brazilian Real. REX was defined as $REX = (NEX * CPIBRA)/CPIUSA$. The data of NEX, CPI_{BR} and CPI_{US} were obtained from the database of the Federal Reserve Bank of St. Louis (2016). All data are seasonally adjusted.

4.- Empirical Models

In this section, first, we will set up the empirical model of VEC with cointegration as the main model of this study. Second, the mathematical structures of NARX and ANFIS models will be shown respectively. These two models are added to the study to compare the results of VEC model with cointegration in testing the validity of the J-curve hypothesis between Brazil and the USA. The empirical model of VEC with cointegration can be written:

$$TB_t = \alpha + \beta PI_{BR,t} + \gamma PI_{US,t} + \delta REX_t + \varepsilon_t \quad (1)$$

The empirical model in Eq. (1) can be expressed in logarithmic form as follows:

$$lnTB_t = \alpha + \beta lnPI_{BR,t} + \gamma lnPI_{US,t} + \delta lnREX_t + \varepsilon_t \quad (2)$$

In Eq. (2) $lnTB_t$, (as the dependent variable of the model) is the rate of Brazil's imports divided by the Brazil's exports. $lnPI_{BR,t}$ and $lnPI_{US,t}$ (as the independent variables) are Brazil's and the USA's Industrial Production Indexes. $lnREX_t$, (as the other independent variable) is the real exchange rate between Brazilian Real and the USD. In VEC model with cointegration, the next following steps will be taken.

First, we apply to unit root test to know whether the series are stationary. If the series are stationary we can apply to the cointegration test giving information about the number of endogenous variables that equal to the number of cointegration vector (Augusto et al. 2005). Augmented-Dickey Fuller (1979) and Philips-Perron (1988) unit root tests will be used.

Second, we estimate the VAR model to determine the optimal number of lags by using the Akaike Info Criterion (AIC), the Final Prediction Error (FPE), the Likelihood Ratio (LR), the Schwarz info criterion (SC) and the Hannan Quinn info criterion (HQ).

Third, we apply the Johannsen cointegration test to evaluate the validity of a long-run relationship between the variables. In this test, the null hypothesis is "there is no cointegration." If there is a long-run relationship, we reject the null hypothesis.

The next step, after detecting cointegration between the variables, is to apply VEC model in order to identify the time path of dynamic adjustments of short-runs. Finally, the last step is to investigate Granger causality relationship between variables (Engle and Granger 1987). VEC model is shown in the following form of Eq. (3).

$$\Delta lnTB_t = \sum_{j=1}^{n1} b_{1j}\Delta lnTB_{t-j} + \sum_{j=1}^{n2} c_{1j}\Delta lnPI_{BR,t-j} + \sum_{j=1}^{n3} d_{1j}\Delta lnPI_{US,t-j} + \sum_{j=1}^{n4} e_{1j}\Delta lnREX_{t-j} + k_1 ECT_{t-1} + \omega_{1t} \quad (3)$$

As far as NARX and ANFIS models are concerned, the variables of these two models have already been defined in cointegration analysis as shown in Eq. (2). On the other hand, the mathematical structure of NARX model can be written:

$$y(t) = f(y(t-1), y(t-2), \dots, y\left(t-n_y\right), u(t-1), u(t-2), \dots, u(t-n_u) \quad (4)$$

When we adapt the same variables of VEC model to Eq. (4) we can write the following equation for NARX model.

$$TB(t) = f(TB(t-1), TB(t-2), \dots, TB(t-n_{TB}), REX(t-1), REX(t-2), \dots, REX(t-n_{REX}), PI_{US}(t-1), PI_{US}(t-2), \dots,, PI_{US}(t-n_{PIUS}), PI_{BR}(t-1), PI_{BR}(t-2), \dots,, PI_{BR}(t-n_{PIBR}) \quad (5)$$

$TB(t)$ is the current prediction value to the input of the network (through delays) since TB is a function of $REX, PI_{BR}\ and\ PI_{US}$. The structure of ANFIS model, testing the validity of J-curve hypothesis between Brazil and the USA, can be shown with the following figure and Eq. (6).

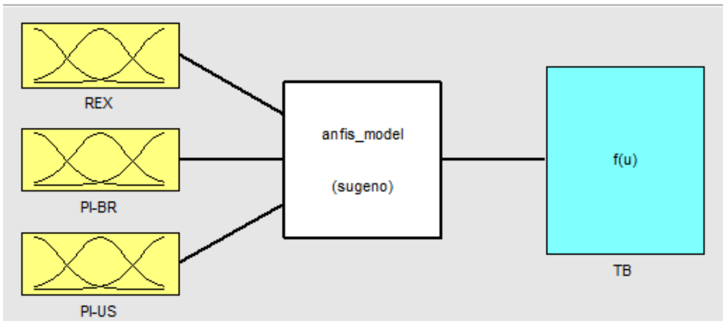

Figure: 1 The Structure of ANFIS Model

In this model, REX, PI_{US}, PI_{BR} are inputs and TB is output. 27 fuzzy “if, then” rules (Eq. 6) are constructed as proposed by Takagi and Sugeno (1985). *Ai*, B_i, C_i are fuzzy sets

$$\begin{cases} R_1: If\ REX\ is\ A_1\ and\ PI_{US}\ is\ B_1\ and\ PI_{BR}\ is\ C_1\ then\ f_1 = p_1 REX + q_1 PI_{US} + r_1 PI_{BR} + k_1 \\ R_2: If\ REX\ is\ A_2\ and\ PI_{US}\ is\ B_2\ and\ PI_{BR}\ is\ C_2\ then\ f_2 = p_2 REX + q_2 PI_{US} + r_2 PI_{BR} + k_2 \\ \vdots \\ R_{27}: If\ REX\ is\ A_{27}\ and\ PI_{US}\ is\ B_{27}\ and\ PI_{BR}\ is\ C_{27}\ then\ f_{27} = p_{27} REX + q_{27} PI_{US} + r_{27} PI_{BR} + k_{27} \end{cases} \quad (6)$$

5.- Empirical Results of VEC with cointegration, NARX and ANFIS Models Respectively

The results of Augmented Dickey Fuller and Philips Perron unit root test for all variables in the empirical model were reported in Table 1.

Table 1: Results of the Unit Root Test of Augmented Dickey Fuller and Philips Perron

Variable	ADF Test Statistics		PP Test Statistics	
	Level	*Intercept*	*Level*	*Intercept*
LTB	-1.921	-2.209	-1.897	-1.971
ΔLTB	-11.163*	-11.123*	-13.695*	-13.638*
LREX	-0.694	-1.935	-0.599	-1.748
ΔLREX	-9.038*	-9.005*	-8.972*	-8.937*
LIPIBRA	0.769	-1.182	0.975	-1.046
ΔLIPIBRA	-9.814*	-9.945*	-10.914*	-11.180*
LIPIUSA	2.018	-0.934	2.802	-0.875
ΔLIPIUSA	-4.855*	-5.335*	-4.495*	-4.847*

ADF Test critical value: *1% -2.582, **5%-1.943 PP Test critical value: *1% -2.582, **5%-1.943 (Level)
ADF Test critical value: *1% -3.478, **5%-2.882 PP Test critical value: *1% -3.478, **5%-2.882 (Intercept)

The results in Table 1 illustrate that the first differences of all the variables are stationary and all variables reveal *I (1)* behavior. In other words, it was observed that all variables are suitable for cointegration analysis. The VAR model was estimated to determine lag length. Optimal lag length was determined as 2 by using FPE and AIC and as 3 by using SC and HQ info criteria. In using the lag number as 3, a residual autocorrelation problem ensued at the 5% level. For this reason, the number of lags was accepted as 2. The results of the Johannsen cointegration test were reported in Table 2.

Table: 2 Results of Johannsen Cointegration Test

Hypothesized No. of CE(s)	Trace Statistic	0.05 Critical Value	Max-Eigen Statistic	0.05 Critical Value
None *	50.46998	40.17493	31.84072	24.15921
At most 1	18.62926	24.27596	10.32740	17.79730
At most 2	8.301865	12.32090	5.353167	11.22480
At most 3	2.948698	4.129906	2.948698	4.129906

* denotes rejection of the hypothesis at 0.05 level

Cointegration Equation: Normalized cointegration coefficients (standard error in parentheses).

	$LREX$	LPI_{US}	LPI_{BR}
LTB	-1.6345	-0.66318	0.36478
1.00	(0.1433)	(0.3999)	(0.2179)

The results of Table 2 indicate that there is a statistically significant cointegration among the variables since test statistics exceeded the critical value. While the long-run coefficient of the *LREX* is negative and elastic (-1.63) the long run coefficient of the LPI_{US} is negative and inelastic (-0.66). On the other hand, the long run coefficient of the LPI_{BR} is positive and inelastic (0.36). The cointegration equation does not contain an intercept or trend and the variables $LREX$, LPI_{US}, LPI_{BR} are statistically significant at 1%, 5% and 10% levels respectively. The results of the VEC model with cointegration were reported in Eq. (7) (*Denotes significant at 5% level, **Denotes significant at 1% level).

$$\Delta(LTB) = -0.25^{**}(ECT) - 0.16^{*}\big(\Delta LTB(-1)\big) - 0.27^{*}\big(\Delta LTB(-2)\big) - 0.11\big(\Delta LREX(-1)\big) - 0.02\big(\Delta LREX(-2)\big) - 2.92^{*}\big(\Delta LPI_{US}(-1)\big) - 0.28\big(\Delta LPI_{US}(-2)\big) + 1.05^{*}\big(\Delta LPI_{BR}(-1)\big) + 0.71\big(\Delta LPI_{BR}(-2)\big) \quad (7)$$

A shock to the *i*th variable does not affect only *i*th but is transmitted to all the other endogenous variables through the dynamic structure of the VAR. The Impulse-Response functions show these relations in Appendix 1.

In Eq. (7) the error correction term (*ECT*) is negative and statistically significant as expected. Short-run coefficients of $\Delta LREX(-1)\ and\ \Delta LREX(-2)$ are not statistically significant. While $\Delta LPI_{US}(-1)\ and\ \Delta LPI_{BR}(-1)$ are statistically significant, $\Delta LPI_{US}(-2) and\ \Delta LPI_{BR}(-2)$ are not. The next step is to calculate the VAR Granger causality relationships between the variables. The results of the VAR Granger causality test were reported in Table 3.

Table: 3 VAR Granger Causality Test Results

	Chi-sq	df	Prob.
$\Delta LTB \rightarrow \Delta LREX$	9.775	2	0.0063
$\Delta LTB \rightarrow \Delta LPI_{US}$	11.257	2	0.0036
$\Delta LTB \rightarrow \Delta LPI_{BR}$	10.969	2	0.0041
$\Delta LPI_{US} \rightarrow \Delta LTB$	5.319	2	0.07

From Table 3, we found a double way Granger causality relationship between ΔLTB and ΔLPI_{US} and a one way Granger causality relationship from $\Delta LTB\ to\ \Delta LREX, \Delta LPI_{BR}\ and\ \Delta LPI_{US}$. The negative relationship between TB and REX, shown in Table 2, confirms that there is no J-curve hypothesis between two countries.

As far as NARX and ANFIS models are concerned, these two models differ from VEC model with cointegration and do not provide equations defining the relationships between the variables. Instead, they give us the responses (elasticities) of *TB* to the 1% and 5% changes in *REX*.

For NARX model analysis, the data set were divided into three sets as training, validating and testing sets. While training set is used by neural network to learn patterns present in the data testing set is used to test whether the network is able to work also on the data that were not used in the previous process. Validating set is used for the final check on the performance of the trained network. If the error is at the acceptable level it is deduced that algorithm learned the relationships between variables. On the other hand, a significant serial correlation (autocorrelation) should not be between the error terms. The estimated model successfully passed all of the testing criteria. The parameters and the network structure of NARX were reported in Table 4. The estimated model is also shown in Eq. 8.

Table 4: NARX Network Structure and Parameters

Network type	NARX
Number of layer	3
Number of hidden layer	1
Number of neuron in hidden layer	10
Number of neuron in output layer	3
Input activation function	Tangent sigmoid
Output activation function	Linear
Epoch	12
Lag length	2
MSE (training)	0,0056
MSE (validating)	0,0057
MSE (Test)	0,0062
R^2	0,956

$$TB(t) = f(TB(t-1), TB(t-2), REX(t-1), REX(t-2), PI_{US}(t-1), PI_{US}(t-2), PI_{BR}(t-1), PI_{BR}(t-2)) \quad (8)$$

In ANFIS model analysis, we used three inputs and one output as already shown in Eq. (6). We constructed the rules for a Takagi-Sugeno type ANFIS model. Similar to the NARX model, there are some criteria in order to test the model's performance and the estimated ANFIS model passed all of the performance testing criteria successfully. After the training process of the estimated ANFIS model, the structure and parameters were reported in Table 5.

Table 5: ANFIS Structure and Parameters

Network type	ANFIS (Sugeno type)
Number of layer	6
Iteration	50
Input membership function	Trapezoidal MF
Output function	Constant
Number of membership function	3-3-3
Number of fuzzy rule	27
Optimization algorithm	Hybrid (Back prob. and LSE)
"and" method	prod
"or" method	probor
Clarification method	wtaver

MSE (training)	0,050
MSE (Test)	0,277
MSE (Check)	0,412

The responses (elasticities) of *TB* to the 1% and 5% changes in *REX* within the NARX and ANFIS models were reported in Figure 2 and 3.

$$NARX\begin{cases} REX\ \%1 \uparrow \rightarrow TB\ \%0{,}99 \downarrow \\ REX\ \%5 \uparrow \rightarrow TB\ \%4{,}24 \downarrow \\ REX\ \%1 \downarrow \rightarrow TB\ \%0{,}87 \uparrow \\ REX\ \%5 \downarrow \rightarrow TB\ \%1{,}76 \uparrow \end{cases}$$

Figure 2: NARX Model

The results of the NARX model indicate that when *REX* increases at 1% and 5%, *TB* decreases at 0.99% and 4.24% respectively. On the other hand, when *REX* decreases at 1% and 5%, *TB* increases at 0.87% and 1.76% respectively. Therefore, the negative relationship between two variables indicates that the J-curve hypothesis is not valid between Brazil and the USA.

$$ANFIS\begin{cases} REX\ \%1 \uparrow \rightarrow TB\ \%\ 0{,}99 \downarrow \\ REX\ \%5 \uparrow \rightarrow TB\ \%\ 4{,}17 \downarrow \\ REX\ \%1 \downarrow \rightarrow TB\ \%\ 0{,}79 \uparrow \\ REX\ \%5 \downarrow \rightarrow TB\ \%\ 1{,}99 \uparrow \end{cases}$$

Figure 3: ANFIS Model

The results of ANFIS model indicate that when *REX* increases at 1% and 5%, *TB* decreases at 0.99% and 4.17% respectively. On the other hand, when *REX* decreases at 1% and 5% *TB,* increases at 0.79% and 1.99% respectively. Therefore, the negative relationship between two variables also indicates that the J-curve j hypothesis is not valid between Brazil and the USA. The 3D graphics of ANFIS model were given in Appendix 2.

The empirical results of these two models in testing the J-curve hypothesis confirm the empirical results of VECM with cointegration. Furthermore, we used Mean Squared Error (MSE) statistics to compare the performances of VEC model with cointegration, NARX and ANFIS models. The MSE statistics were reported in Table 6.

Table 6: Model Performance

Model	MSE
VECM with cointegration	0.02
NARX	0.0057
ANFIS	0.140

Under the consideration of MSE statistics, NARX with the lowest MSE coefficient was found to be the most successful model for the estimation between TB and REX. In other words, the NARX model performed better than the other two models in the estimation of two variables.

6.- Conclusion Remarks

This study aims to investigate the validity of J-curve hypothesis between Brazil and the USA for the period of 1981Q1- 2015Q1.To achieve this aim, VEC with cointegration, NARX and ANFIS testing models were applied separately to strengthen this investigation of the J-curve hypothesis between two countries. NARX and ANFIS, as ANN models, were used for the first time for testing the validity of the J-curve hypothesis. The empirical test results of all models examined indicate that the J-curve hypothesis is not valid between Brazil and the USA. The real depreciations of the Brazilian currency against the USD do not make a positive contribution to the trade balance of Brazil in which different exchange rate regimes were adopted by the Central Bank of Brazil. The effects of different exchange rate regimes and changing values of Brazilian currency in terms of appreciations, deprecations and devaluations remain an important economic issue for the country. Therefore, based on this importance, it is suggested that the future studies should apply to different methods and models to test the validity of the J-curve hypothesis between two countries. Because, Brazil's long experience with regards to adoptions of different exchange rate regimes and devaluations can be offered as a perfect example for other developing countries. As far as the methodological performances of the models examined in this study are concerned, NARX performed better than ANFIS and VEC with cointegration models in estimation between the real exchange rate (REX) and trade balance (TB) between two countries.

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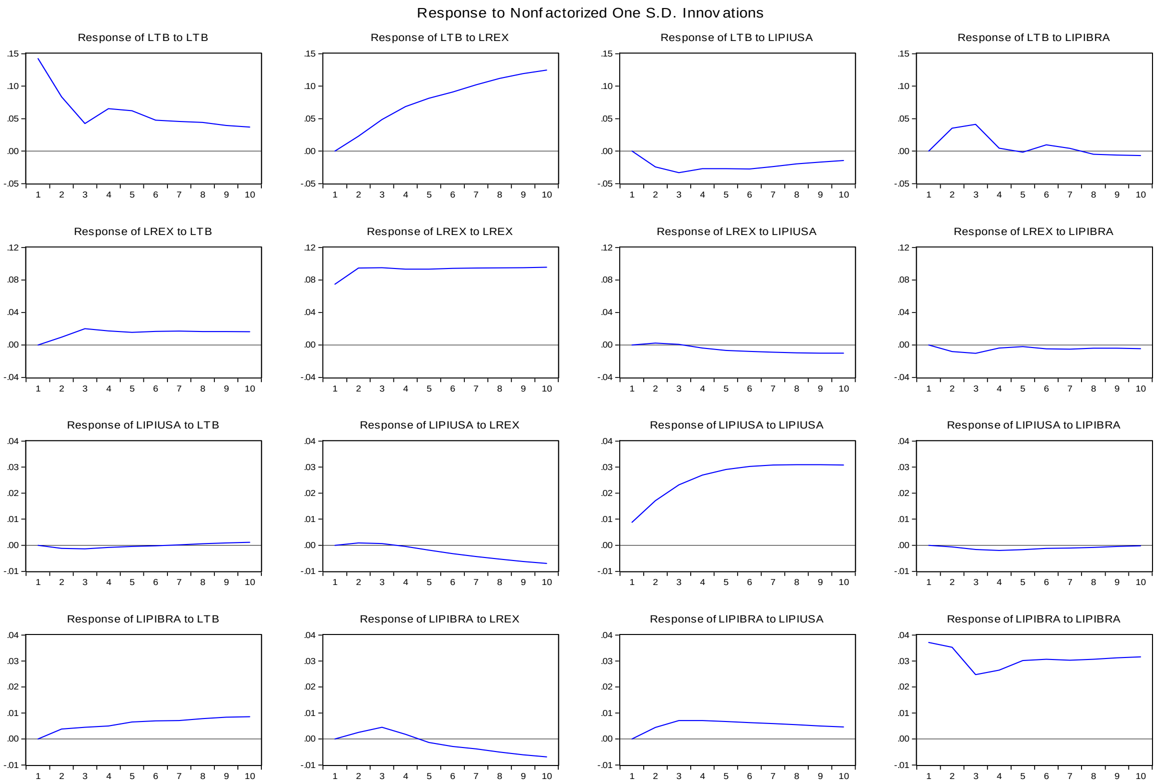
Appendix 1

Impulse Response Results



Appendix 2: Relationships between Variables (3D Graphics)