MULTIVARIATE RISK-RETURN DECISION MAKING WITHIN DYNAMIC ESTIMATION

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ABSTRACT

Risk management in this paper is focused on multivariate risk-return decision making assuming time-varying estimation. Empirical research in risk management showed that the static "mean-variance" methodology in portfolio optimization is very restrictive with unrealistic assumptions. The objective of this paper is estimation of time-varying portfolio stocks weights by constraints on risk measure. Hence, risk measure dynamic estimation is used in risk controlling. By risk control manager makes free supplementary capital for new investments.

Univariate modeling approach is not appropriate, even when portfolio returns are treated as one variable. Portfolio weights are time-varying, and therefore it is necessary to reestimate whole model over time. Using assumption of bivariate Student's t-distribution, in multivariate GARCH(p,q) models, it becomes possible to forecast time-varying portfolio risk much more precisely. The complete procedure of analysis is established from Zagreb Stock Exchange using daily observations of Pliva and Podravka stocks.

RESUMEN

El manejo del riesgo es enfocado en este trabajo como la toma de decisión sobre la estimación de un riesgo del retorno multivariado asumiendo variabilidad en el tiempo.. Investigaciones empíricas del manejo del riesgo han mostrado que la metodología estática de "media-varianza" en la optimización de portafolio es muy restrictiva bajo asunciones poco realistas. El objetivo de este trabajo es la estimación de pesos para los portafolios de acciones con variación en el tiempo con restricciones sobre la medida de riesgo. Entonces, la estimación dinámica de la medida de riesgo es usada en el control del riesgo. Controlando el riego el manager libera capital suplementario para nuevas inversiones.

El enfoque univariado no es apropiado, aun cuando los retornos de portafolio sean tratados como una variable. Los pesos para los portafolios varían en el tiempo, por tanto es necesario re-estimar todo el modelo en el tiempo. Asumiendo una distribución T-Student bivariada, en el modelo de GARCH(p, q) mutivariado, es posible predecir el riesgo del portafolio que varia en el tiempo mucho mas precisamente. El procedimiento completo de análisis es desarrollado para el Zagreb Stock Exchange usando las observaciones diarias de los stocks de Pliva y Podravka.

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1. INTRODUCTION

The aim of this paper is estimation of time-varying stocks weights in portfolio optimization using multivariate approach. The purpose of this approach, realized using multivariate GARCH(p,q) model, is forecasting of time-varying conditional expectation and conditional variance. Those forecasted values of first and second order moments are inputs for portfolio return maximization by constraints on standard deviation as risk measure.

The assumption of bivariate Student's t-distribution is used in maximization of the likelihood function to estimate parameters of DVEC-GARCH(1,1) model with Cholesky factorization. Sample properties, especially leptokurtosis of distribution with fat tails, shows that assumption of Student distribution is the most appropriate (Arnerić, Jurun, Pivac 2007).

Modern financial analysis requires forecasting the dependences in the second order moments of portfolio returns. Forecasting is necessary, apart from other reasons, because financial volatilities moves together over time across assets and markets.

According to these requirements multivariate modeling framework leads to more relevant empirical models in comparison to estimations based on separate univariate models. This appears from the practical need for options pricing, portfolio selection, hedging and Value-at-Risk estimation. Furthermore, a fundamental characteristic of modern capital markets leads to application of multivariate volatility models. This can be summarized in following facts:

- the volatility of one market leads to volatility of other markets;
- the volatility of an stock transmits to another stock;
- the correlation between stock returns change over time;
- the correlation between stock returns increases in the long run because of the globalization of financial markets.

In order to respect all these facts and modern financial requirements as the most useful methodological tool financial econometrics offers Multivariate Generalized Autoregressive Conditional Heteroscedasticity models - MGARCH (Bauwnes, Laurent, Romboust 2006).

The complete procedure of analysis is established using daily observations of Pliva and Podravka stocks, as the most frequently traded stocks from CROBEX index at Zagreb Stock Exchange. Data on observed stocks was provided by Croatia capital market. In this paper daily data of compound returns of named stocks from 1st January 2005 to 16th October 2007 are used (www.zse.hr).

This paper is organized as follows. After introduction, assumption of the bivariate Student's t-dustribution is presented. Next section presents the methodology employed in MGARCH model selection. Following part gives estimation of conditional variances and correlation. The topic of the next section is optimal dynamic portfolio weights forecasting. The final section is dedicated to conclusion remarks.

2. ASSUMPTION OF THE BIVARIATE STUDENT'S T-DISTRIBUTION

Returns from financial instruments such as exchange rates, equity prices and interest rates measured over short time intervals, i.e. daily or weekly, are characterized by high kurtosis. In practice, the kurtosis is often larger than three, leading to estimation of non-integer degrees of freedom. Thus, degrees of freedom can easily be estimated using the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm. BFGS is a method to solve an unconstrained nonlinear optimization problem (Bazarra, Sherali, Shetty 1993)

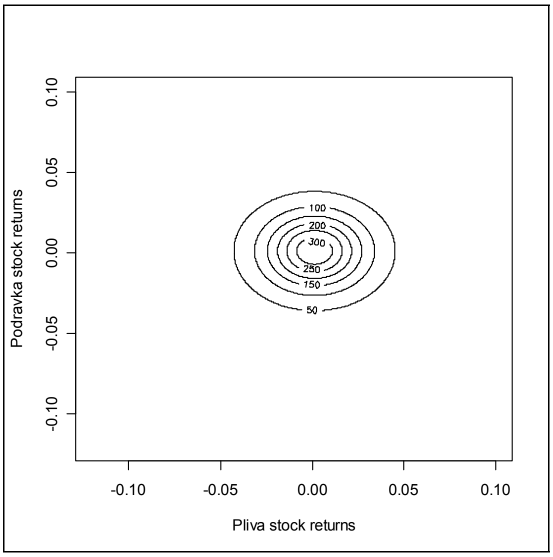
The BFGS method is derived from the Newton's optimization methods, as a class of hill-climbing optimization techniques that seeks the stationary point of a function, where the gradient is zero. Newton's method assumes that the function can be locally approximated as a quadratic in the region around the optimum, and use the first and second derivatives to find the stationary point.

In Quasi-Newton methods the Hessian matrix of second derivatives of the function to be optimized does not need to be computed at any stage. The Hessian is updated by analyzing successive gradient vectors instead. The BFGS method is one of the most successful algorithms of this class. In this paper it will be used in bivariate Student's t-distribution degrees of freedom estimation by maximization of the likelihood function.
The standardized multivariate Student density from the k-dimensional vector of returns **X** is given as:

$$f(X)=\frac{\Gamma\left(\frac{df+k}{2}\right)}{\Gamma\left(\frac{df}{2}\right)\left(\sqrt{\pi(df-2)}\right)^{k}}\left(1+\frac{\left(X^{T}X\right)^{2}}{df-2}\right)^{-\frac{k+df}{2}}, \tag{1}$$

where $\Gamma(\cdot)$ is gamma function and df is a shape parameter - degrees of freedom.

Figure 1: Contour plot of the Bivariate Student's t-distribution Density ($df = 3.2$)



Source: According to data on ZSE

It is imposed that df is larger than 2 to ensure the existence of the variance matrix. There are no mathematical reasons why the degrees of freedom should be an integer, however when $df \to 2$ the tails of the density become heavier (Heikkinen, Kanto 2002). Hence, the assumption that returns are independently and identically normally distributed is unrealistic.

According to BFGS algorithm degrees of freedom of the bivariate Student t-distribution for Pliva and Podravka stock returns are estimated at level 3.2. Standard error of estimated degrees of freedom is 0.200705, which means that parameter distribution is statistically significant. Figure 1 shows elliptic contour plot of the bivariate Student's t-distribution density with estimated degrees of freedom. Namely, as contour of bivariate Student's t-distribution density is more elliptic, the less are estimated degrees of freedom (Bauwnes, Laurent 2005).

The Student density has become widely used due to it's simplicity and it's inherent ability to fit excess kurtosis.

3. APPROPRIATE MULTIVARIATE GARCH(p,q) MODEL SELECTION

Typical example when it is necessary to simultaneously observe return movements of several stocks is market portfolio return. If market portfolio consists of *k* stocks, expected return ($r_{P,t}$) and variance of complete portfolio ($\sigma^2_{P,t}$) can be defined in the following way:

$$\begin{aligned} r_{P,t} &= w^T \mu_t \\ \sigma^2_{P,t} &= w^T \Sigma_t w \end{aligned}, \qquad (2)$$

where w is the vector of portfolio weights, μ_t is the vector of expected returns and Σ_t is the variance-covariance matrix of the returns.

According to equation (2) it's obvious that the mean vector μ_t and the variance-covariance matrix Σ_t are not constant over time. Univariate GARCH(p,q) model for portfolio variance can be estimated for a given weights vector. Because the weights vector changes over time, it is necessary to estimate the model again and again. That's way it is wisely to introduce the possibilities of MGARCH model. If a MGARCH model is fitted, the multivariate distribution of the returns can be directly used to compute time varying mean vector and variance-covariance matrix of the returns. Hence there is no need to reestimate the model for a different weights vector.

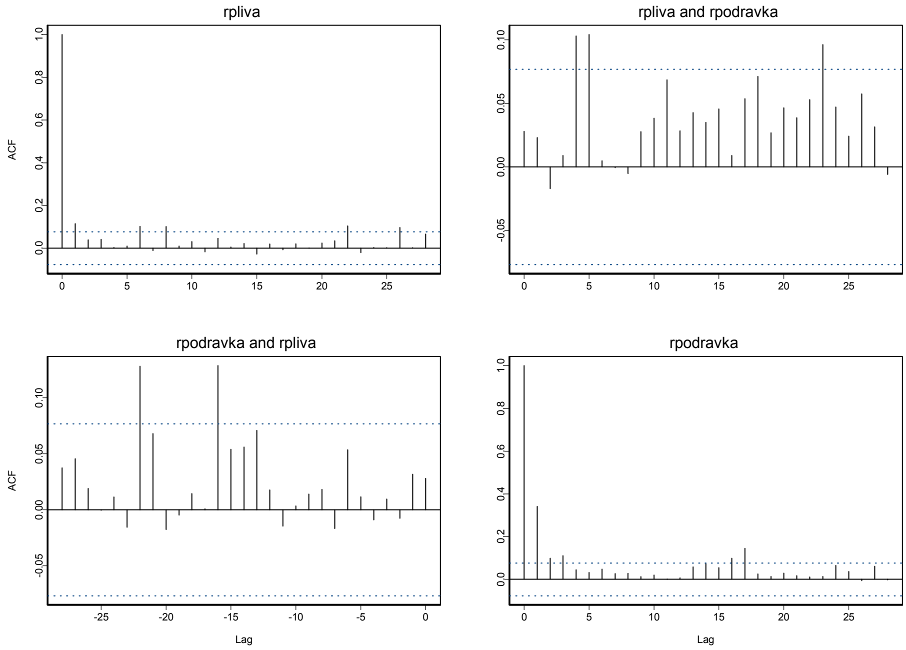
Consider a bivariate vector stochastic process $\{r_t\}$ of dimension 2×1. Dynamic model with time varying means, variances and covariances for 2 component can be denoted by:

$$\begin{aligned} r_t &= \mu_t + \varepsilon_t \\ \mu_t &= E(r_t | I_{t-1}) \\ \varepsilon_t &= \Sigma_t^{1/2} u_t \\ \Sigma_t &= V(r_t | I_{t-1}) \end{aligned} \qquad (3)$$

In above relations μ_t is mean vector conditioned on past information, and Σ_t is conditional variance matrix. Furthermore, it is assumed that expected value of u_t is null-vector with variance equal to identity matrix, as well as that variance Σ_t is positive definite matrix. This assumption is not generally satisfied automatically. Positive definite variance-covariance matrix can be obtained by the Cholesky factorization of Σ_t (Tsay, 2005). Before the concrete modeling procedure it is useful to estimate autocorrelation and partial autocorrelation coefficients of squared returns.

Therefore correlograms and cross-correlograms are presented in Figure 2 for thirty time lags.
It is obvious from Figure 2 that estimated autocorrelation coefficients are statistically significant in many time lags when exceed critical value presented by dotted lines. This confirms hypothesis that observed return time series contain ARCH effects (heteroscedasticity).

Figure 2: Correlograms and cross-correlograms between Pliva and Podravka square returns



Source: According to data on www.zse.hr

According to given assumptions a diagonal VEC model with Cholesky factorization is estimated. Univariate GARCH(1,1) model can be generalized into bivariate, as folows:

$$vech(\Sigma_t) = C + Avech(\varepsilon_{t-1}\varepsilon_{t-1}^T) + Bvech(\Sigma_{t-1}) \qquad (4)$$

In equation (4) $vech(\cdot)$ operator denotes the column-stacking operator applied to the lower portion of the symmetric matrix. The model (4) requires the estimation of 21 parameters (C matrix has 3 elements, A and B matrices each 9 elements).

To overcome dimensionality problem in VEC methodology A and B matrices can be restricted to diagonal elements. Therefore, bivariate DVEC-GARCH(1,1) model is restricted to $3(k+1) = 9$ parameters. Furthermore, DVEC model can be simplified by restricting A and B to be a vectors or a positive scalars (Zivot, Wang 2006).

Table 1. VAR(1)-DVEC(1,1) model with Cholesky factorization

```
Estimated Coefficients:
--------------------------------------------------------------
                  Value Std.Error t value Pr(>|t|)
         C(1) 0.0005129 0.0003415  1.5019 0.066800
         C(2) 0.0004966 0.0004663  1.0649 0.143660
  AR(1; 1, 1) 0.1047563 0.0390424  2.6831 0.003740
  AR(1; 2, 2) 0.0508917 0.0448776  1.1340 0.128605
      A(1, 1) 0.0036373 0.0002112 17.2239 0.000000
      A(2, 1) 0.0002860 0.0004458  0.6416 0.260690
      A(2, 2) 0.0045237 0.0004084 11.0764 0.000000
ARCH(1; 1, 1) 0.7506377 0.0234564 32.0013 0.000000
ARCH(1; 2, 1) 0.1746712 0.0569743  3.0658 0.001131
ARCH(1; 2, 2) 0.4859494 0.0323898 15.0031 0.000000
  GARCH(1; 1) 0.7753328 0.0133463 58.0935 0.000000
  GARCH(1; 2) 0.8275629 0.0157466 52.5551 0.000000
--------------------------------------------------------------
Information criteria:
--------------------------------------------------------------
AIC(12) = -7406.841
BIC(12) = -7353.080
```

Source: According to data on www.zse.hr

A disadvantage of the DVEC model is that there is no guarantee of a positive definite variance-covariance matrix. Hence, different DVEC(1,1) models with different restrictions on coefficients matrices will be analyzed, undertaking Cholesky factorization.

For complete estimation procedure it is necessary to estimate mean vector of returns using VAR(1) system. Also in vector autoregression model of order 1, only diagonal coefficients will be estimated. Estimation results of VAR(1)-DVEC(1,1) model with Cholesky factorization are presented in Table 1.

According to Table 1, in which matrix A is restricted to be diagonal and matrix B a vector, the results are presented as follows:

$$\begin{bmatrix} r_t^{PL} \\ r_t^{PD} \end{bmatrix} = \begin{bmatrix} 0.000513 \\ 0.000497 \end{bmatrix} + \begin{bmatrix} 0.104756 & \\ & 0.05089 \end{bmatrix} \cdot \begin{bmatrix} r_{t-1}^{PL} \\ r_{t-1}^{PD} \end{bmatrix}$$

$$\Sigma_t = \begin{bmatrix} 0.000013 \\ 0.000001 \\ 0.000021 \end{bmatrix} + \begin{bmatrix} 0.563457 \\ 0.131112 \\ 0.266656 \end{bmatrix} \cdot vech(\varepsilon_{t-1}\varepsilon_{t-1}^2) + \begin{bmatrix} 0.601141 \\ 0.641637 \\ 0.684860 \end{bmatrix} \cdot vech(\Sigma_{t-1}) \tag{5}$$

In Table 2 the results of normality test, Ljung-Box test and Lagrange multiplier test are presented, indicating that in estimated model (5) remains no autocorrelation and no ARCH effects, i.e. there is no heteroscedasticity left.

Table 2: Diagnostic tests of standardized residuals

```
Normality Test:
--------------------------------------------------------------
          Jarque-Bera P-value Shapiro-Wilk    P-value
   rpliva      1779.3       0       0.9114 0.000e+000
rpodravka       704.7       0       0.9619 4.441e-016

Ljung-Box test for standardized residuals:
--------------------------------------------------------------
          Statistic P-value Chi^2-d.f.
   rpliva     10.55  0.5677         12
rpodravka     10.17  0.6010         12

Ljung-Box test for squared standardized residuals:
--------------------------------------------------------------
          Statistic P-value Chi^2-d.f.
   rpliva     9.737  0.6390         12
rpodravka     7.282  0.8385         12

Lagrange multiplier test:
--------------------------------------------------------------
           TR^2 P-value F-stat P-value
   rpliva 8.810  0.7191 0.8121  0.7380
rpodravka 6.468  0.8907 0.5940  0.9269
```

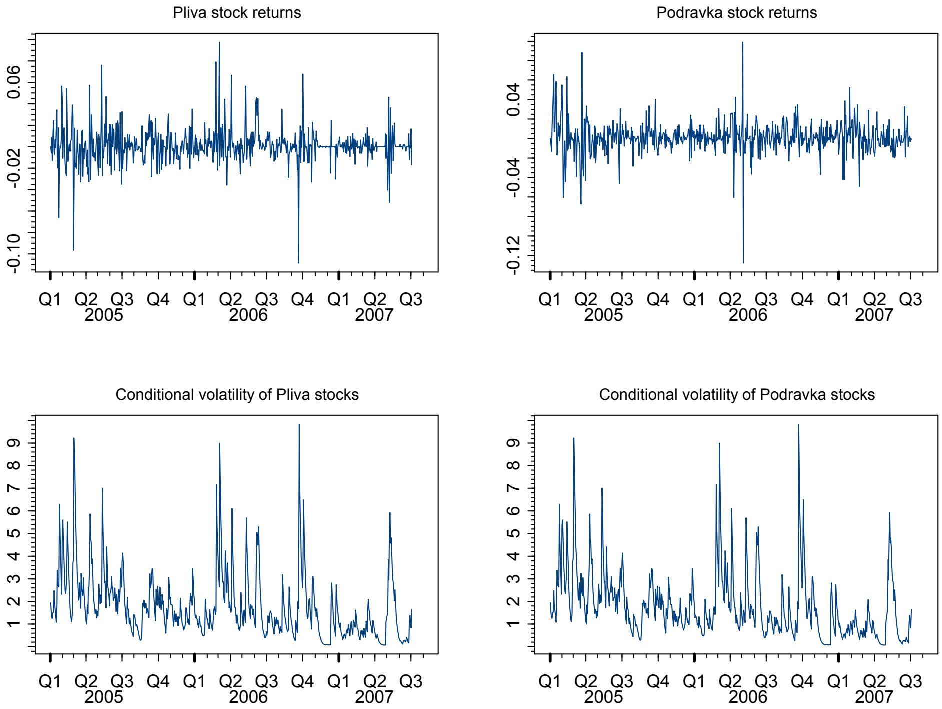
Source: According to data on www.zse.hr

4. ESTIMATION OF CONDITIONAL VARIANCES AND CORRELATION

On Figure 3 there is evident similarity between Pliva and Podravka stock return movements. It can be seen increase in volatility movement in last quarter of 2006. The main precondition for this rise was an intensive increase of investment in investment funds. Especially the increase of investment in the stock funds, caused by increased public awareness of the advantages of such investment (above average yield, higher interest rates offered by banks) as well as the broadening of market supply allowing better risk diversification, has significantly affected the trading volume on the Croatian capital market and thus the volatility of Pliva and Podravka stocks.

These market movements are characteristic of the emerging capital market.

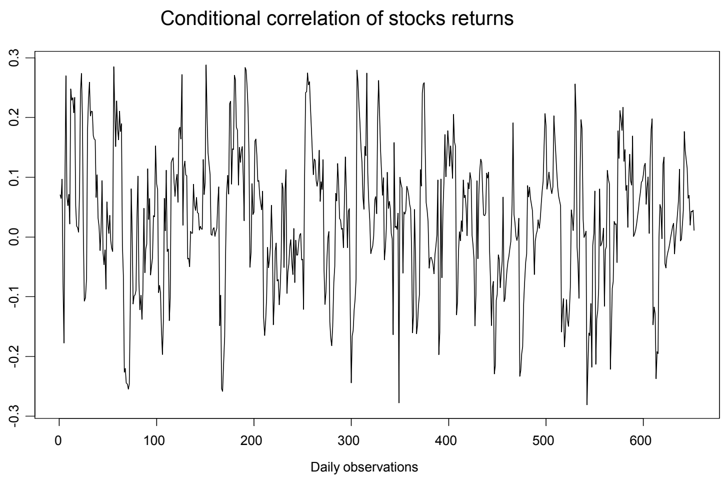
Figure 3: Returns of Pliva and Podravka stocks with their conditional volatilities



Source: According to data on www.zse.hr

Therefore, positive correlation between them can be expected. Considering that correlation is conditioned on past information, it is time-varying. Namely, correlation coefficients are calculated using data of covariances and variances of stock returns in matrix Σ_t. Data are time-varying as it is shown on Figure 4.

Figure 4: Conditional correlation of Pliva and Podravka stock returns



Source: According to data on www.zse.hr

It is obvious that conditional correlation values of stock returns are positive in most trading days. This may be an indicator of emerging bull market.

5. FORECASTING OPTIMAL DYNAMIC PORTFOLIO

According to estimated model (5) predicted values of mean vector and variance-covariance matrix are computed and presented in Table 3 for 10 days-ahead.

Table 3: Ten-days prediction of returns, standard deviations, and correlation between Pliva and Podravka stocks

Predicted Pliva returns	Predicted Podravka returns	Predicted Pliva S.dev.	Predicted Podravka S.dev.	Predicted correlation
-0.0012574771	0.0004609494	0.01877445	0.00928619	0.040395556
0.0003811753	0.0005200320	0.02058464	0.01012907	0.031090784
0.0005528344	0.0005230389	0.02251005	0.01087058	0.024723091
0.0005708168	0.0005231919	0.02456287	0.01153195	0.020176599
0.0005727006	0.0005231997	0.02675579	0.01212782	0.016816266
0.0005728979	0.0005232001	0.02910213	0.01266881	0.014258517
0.0005729186	0.0005232001	0.03161594	0.01316294	0.012261221
0.0005729207	0.0005232001	0.03431216	0.01361647	0.010666151
0.0005729210	0.0005232001	0.03720672	0.01403442	0.009366925
0.0005729210	0.0005232001	0.04031662	0.01442085	0.008290239

Source: According to data on www.zse.hr

Using results from Table 3 in equation (2) vector μ_t and matrix Σ_t are obtained in following optimization problem:

$$
\begin{aligned}
&\max\left(w_{i,t}^T \mu_t\right) \quad i = 1,2 \\
&\sqrt{w_{i,t}^T \Sigma_t w_{i,t}} \le SD_{p,t}^* \\
&\sum_{i=1}^{2} w_{i,t} = 1 \\
&w_{i,t} \quad \forall i
\end{aligned}
\tag{6}
$$

In accordance with optimization problem in (6) dynamic portfolio selection is defined on daily basis, where $SD_{p,t}^*$ is desired portfolio standard deviation in moment t . In this paper optimization is done under assumption of constant risk preference at level of 2% and 3%. Maximization problem (6) is nonlinear programming problem solved using Solver by generalized reduced gradient method (Rombouts, Rengifo 2004).

Ten days forecast results are given in Table 4.

Table 4: Ten days forecast optimal portfolio weights at the standard deviation level of 2% and 3%

k-days forecast		*Stock weights of optimal portfolio selection*									
		1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
$SD_{p,t} \leq 0{,}03$	$w_{plv,t}$	0	0	1	1	1	1	0,9484	0,8723	0,8022	0,7374
	$w_{pod,t}$	1	1	0	0	0	0	0,0516	0,1277	0,1978	0,2626
$SD_{p,t} \leq 0{,}02$	$w_{plv,t}$	0	0	0,8854	0,8074	0,7359	0,6700	0,6094	0,5534	0,5019	0,4545
	$w_{pod,t}$	1	1	0,1146	0,1927	0,2641	0,3300	0,3906	0,4466	0,4981	0,5455

Source: According to data on www.zse.hr

From Table 4 it can be seen that portfolio weights change daily with constant risk preference. It is obvious that portfolio weights are changing at the third day, and more intensively in given time frame at lower risk level (2%). In other words, if investor is willing to accept higher risk his portfolio weights would changed after sixth day (risk level 3%).

According to presented data in Tables 3 and 4 it can be concluded that stock with higher return has higher weight in optimal portfolio. If we compare 9th and 10th prognostic days portfolio is chosen according to defined standard deviation level. Namely, regardless to equal stocks returns higher weight is associated to the stock with lower risk measure.

6. CONCLUSION

This paper is focused on dynamic estimation of stocks weights in portfolio optimization by multivariate approach. For the purpose of the risk controlling it is important to forecast the rate of return and its variance over the holding period and to estimate the risk associated with holding a particular asset.

By risk control manager makes free supplementary capital for new investments. In comparison to the conventional approach, empirical research in risk management showed that the static "mean-variance" methodology is very restrictive with unrealistic assumptions. So, multivariate GARCH(p,q) volatility modeling is more adequate for the purpose of forecasting of time-varying conditional first and second order moments. From MGARCH family models, DVEC(1,1) model with restrictions to positive-definite covariance matrix was used.

The complete procedure of analysis is established using daily observations of Pliva and Podravka stocks, as the most frequently traded stocks from CROBEX index at Zagreb Stock Exchange.
The analysis shows that stocks with negative predicted returns have no weight in portfolio regardless investor risk preference. Stocks with positive predicted returns will have relevant weights in portfolio taking into account profit maximizing with proposed risk measure, i.e. standard deviation. Dynamic portfolio optimization for ten days forecast is solved as nonlinear programming problem by generalized reduced gradient method.

Finally, respecting different investor risk preference at chosen standard deviation level up to 2% and 3% forecasts of optimal portfolio weights for ten days out of sample are made.

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